1.3

New Functions From old Functions

In this section we start with The basic function and obtain New functions by

1-Combining Function

We look at the main ways functions are combined or transformed to form new functions.

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- **Combining functions Algebraically**

جمع
طرح
ضرب

1- $\left(f \frac{+}{\times} g\right)(x) = f(x) \frac{+}{\times} g(x)$

** $D\left(f \frac{+}{\times} g\right) = Df \cap Dg$ المجال هو تقاطع مجال الدالتين

قسمة

2- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$

** $D\left(\frac{f}{g}\right) = Df \cap Dg$ ما عدا $\{g(x) = 0\}$

المجال هو تقاطع مجال الدالتين ما عدا أصفار المقام.

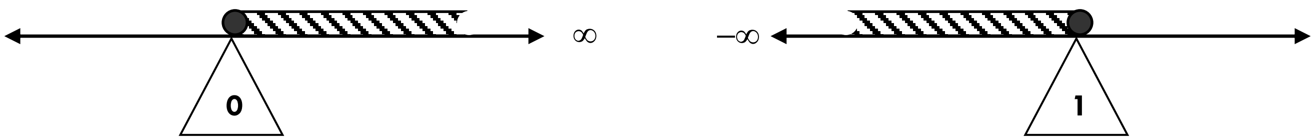
Ex-If: $f(x) = \sqrt{x}$ **and** $g(x) = \sqrt{1-x}$

Find: (1) $(f + g)(x)$ (2) $(f - g)(x)$

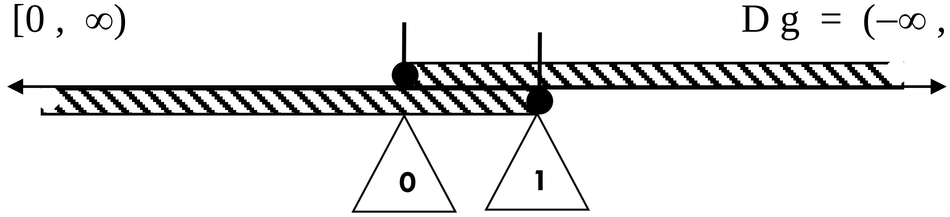
(3) $(f \cdot g)(x)$ (4) $\left(\frac{f}{g}\right)(x)$

and Domain of each.

Solution



$D f = [0, \infty)$ $D g = (-\infty, 1]$



(1) $(f + g)(x) = f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$

(2) $(f - g)(x) = f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$

(3) $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{x} \cdot \sqrt{1-x} = \sqrt{x - x^2}$

• $D\left(f \frac{+}{\times} g\right) = D f \cap D g = [0, \infty) \cap (-\infty, 1] = [0, 1]$

(4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{1-x}}$

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- $D\left(\frac{f}{g}\right) = D f \cap D g - \{$ أصفار المقام $\} = [0 , 1)$

Ex-If: $f(x) = 3x - 3$ & $g(x) = x^2 - 3x - 4$

Find: (1) $f + g$ (2) $f - g$

(3) $f . g$ (4) $\frac{f}{g}$

and Domain of each.

Solution

(1) $(f + g)(x) = (3x - 3) + (x^2 - 3x - 4) = x^2 - 7$

(2) $(f - g)(x) = (3x - 3) - (x^2 - 3x - 4) = -x^2 + 6x + 1$

(3) $(f . g)(x) = (3x - 3) . (x^2 - 3x - 4) = 3x^3 - 12x^2 - 3x + 12$

- $D\left(f \frac{+}{\times} g\right) = D f \cap D g = R \cap R = R = (-\infty , \infty)$

(4) $\left(\frac{f}{g}\right)(x) = \frac{3x - 3}{x^2 - 3x - 4}$

أصفار المقام

$x^2 - 3x - 4 = 0$, $(x - 4)(x + 1) = 0$, $\therefore x = 4$ & $x = -1$

- $D\left(\frac{f}{g}\right) = D f \cap D g - \{4 , -1\}$

$= R - \{4 , -1\} = (-\infty , -1) \cup (-1 , 4) \cup (4 , \infty)$

2-Composition of Functions
تحصيل الدوال

If: f and g are functions , The composition functions

* $(f \circ g)(x) = f(g(x))$

* D $(f \circ g)$ = D الناتج $\cap$ D g الدالة الثانية

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Ex-If: $f(x) = \sqrt{x - 1}$ and $g(x) = x^2 + 1$

1-Find $(f \circ g)(x) = f(g(x))$

نعوض بكل الدالة الأخيرة مكان كل x في الدالة الأولى

$= \sqrt{x^2 + 1 - 1}$

$= |x|$

2- Find $(g \circ f)(x)$

نعوض بكل الدالة الأخيرة مكان كل x في الدالة الأولى

$= (\sqrt{x - 1})^2 + 1$

$= x - 1 + 1$

$= x$

if: $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = \sin x$

1- $(f \circ g)(x) = (\sqrt{x})^2 = x$

2- $(h \circ g)(x) = \sin \sqrt{x}$

3- $(f \circ h)\left(\frac{\pi}{6}\right)$

بالتعويض بـ 30 في دالة h ثم الناتج يعوض به في دالة f

$$(\sin 30)^2 = \left(\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)$$

4-$(f \circ h)\left(\frac{\pi}{3}\right)$ $\frac{\pi}{3} = 60$

بالتعويض بـ 60 في دالة h ثم الناتج يعوض به في دالة f

$$(\sin 60)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

5 - $(f \circ g \circ h)(x)$

بالتعويض بالدالة h مكان كل x في الدالة g ثم التعويض بالناتج مكان كل x في دالة f

$$\sqrt{\sin x}^2 \longleftarrow \sqrt{\sin x}$$

$\therefore$ = الناتج النهائي $\sin x$

6-$(g \circ f)(4)$

بالتعويض بالعدد 4 في دالة f ثم الناتج يعوض به في دالة g

$\therefore$ = الناتج النهائي 4

If: $f(x) = \cos^2(x + 2)$ Find f , g and h

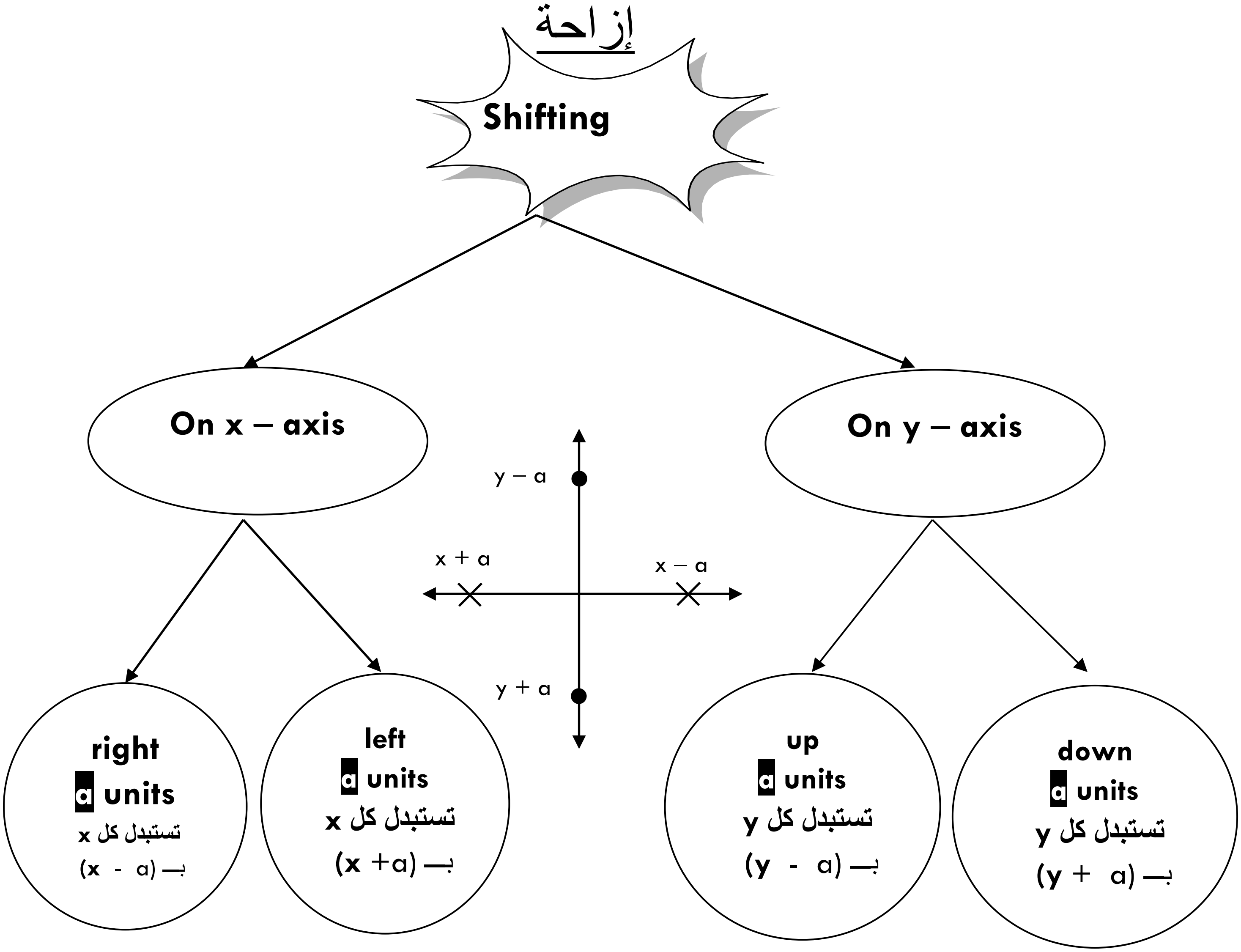
Where $f = f \circ g \circ h$

المعطى هو ناتج التحصيل والمطلوب هو إيجاد الدوال الثلاثة

$f(x) = x^2$, $g(x) = \cos x$, $h(x) = x + 2$

3-Shifting

إزاحة



Find an equation for shifted to the new position:

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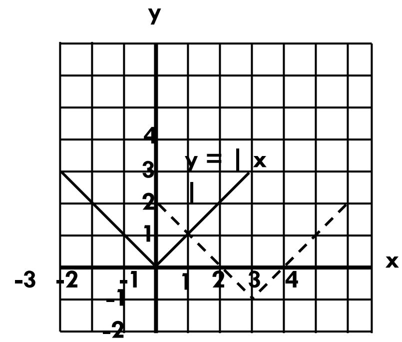
(1) $y = |x|$ $x - 3$ ∴ 3 تحركت يمين x

$y + 1 = |x - 3|$ $y + 1$ ∴ 1 تحركت لأسفل y

$y = |x - 3| - 1$



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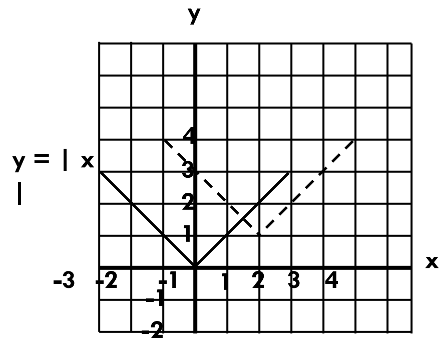
(2) $y = |x|$ $x - 2$ ∴ 2 تحركت يمين x

$y - 1 = |x - 2|$ $y - 1$ ∴ 1 تحركت لأعلى y

$y = |x - 2| + 1$



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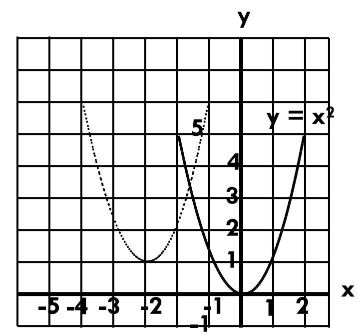
(3) $y = x^2$ $x + 3$ ∴ 3 تحركت يسار x

$y - 1 = (x + 3)^2$ $y - 1$ ∴ 1 تحركت لأعلى y

$y = (x + 3)^2 + 1$



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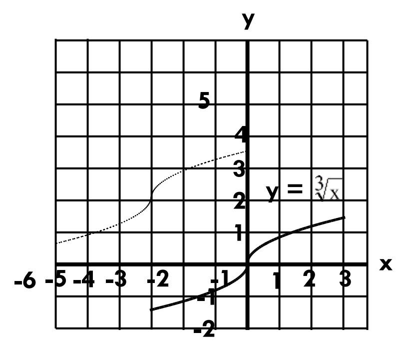
(4) $y = \sqrt[3]{x}$ $x + 3$ ∴ 3 تحركت يسار x

$y - 2 = \sqrt[3]{x + 3}$ $y - 2$ ∴ 2 تحركت لأعلى y

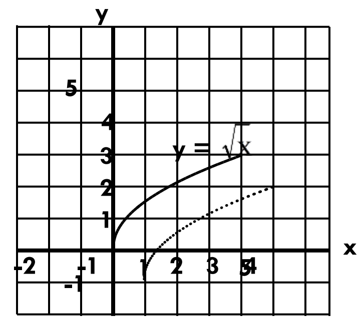
$y = \sqrt[3]{x + 3} + 2$



(5) $y = \sqrt{x}$ x − 1   ∴ x تحركت يمين 1

$y + 1 = \sqrt{x-1}$ y + 1   ∴ y تحركت لأسفل 1

$y = \sqrt{x-1} - 1$

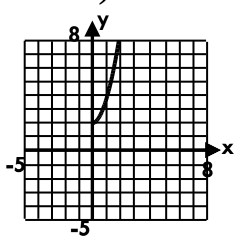


Choose the domain of function $y = \sqrt{x-2}$

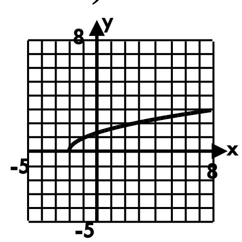
a) $[2, \infty)$ b) $(2, \infty)$ c) $[-2, \infty)$ d) $(0, \infty)$

Choose the correct of function.

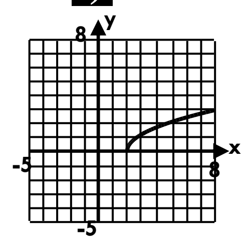
a)



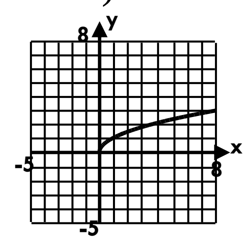
b)



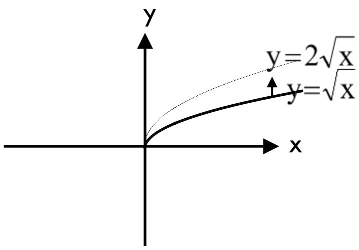
c)



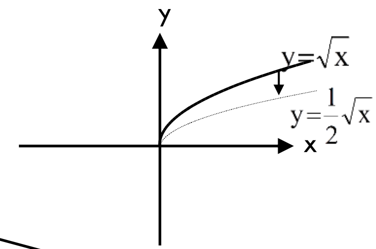
d)



4-Compression and Stretch



Vertical



Stretch by c

تستبدل y بـ $\frac{y}{c}$

Compress by c

$cy = f(x)$

تستبدل y بـ cy

Example

For the function

$y = x^2 - 1$

ind the equation for stretch vertical by a factor of **2**

Solution

$\frac{y}{2} = x^2 - 1$

$y = 2x^2 - 2$

Example

For the function

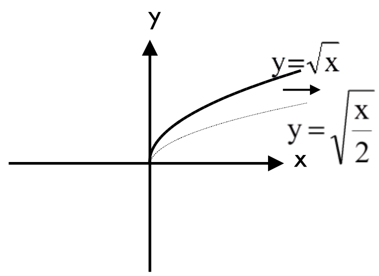
$y = 6x^2 - 1$

find the equation for compress vertical by a factor o **6**

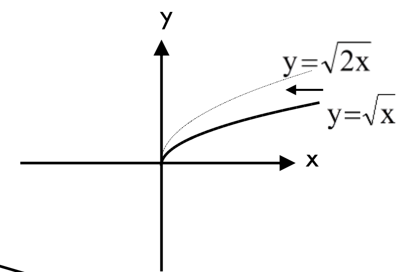
Solution

$6y = 6x^2 - 1$

$y = x^2 - \frac{1}{6}$



Horizontal



Stretch by c

$$y = f\left(\frac{x}{c}\right)$$

استبدل **x** بـ $\frac{x}{c}$

Example

If $y = x^2 - 1$
Stretch horizontal by a factor of **4**

Solution

The new function is

$$y = \left(\frac{x}{4}\right)^2 - 1$$

$$y = \frac{x^2}{16} - 1$$

Compress by c

$$y = f(c\,x)$$

استبدل **x** بـ **c x**

Example

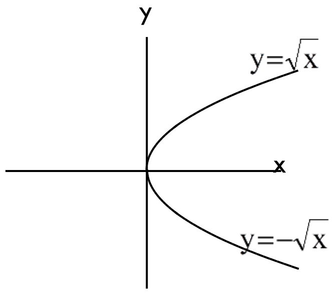
If $y = x^2 - 1$
Compress horizontal by a factor of **3**

Solution

The new function is

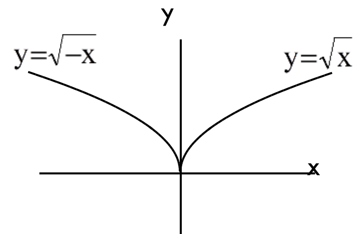
$$y = (3x)^2 - 1$$

$$y = 9x^2 - 1$$



انعكاس الدالة حول محور x

Reflected



انعكاس الدالة حول محور y

about x − axis

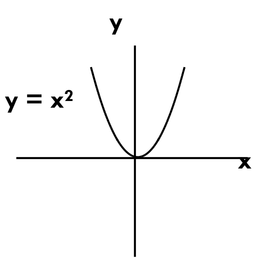
$y = f(x)$

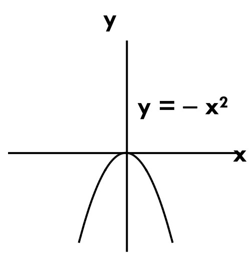
نستبدل y بـ − y

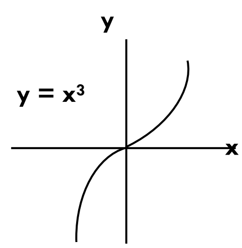
about y − axis

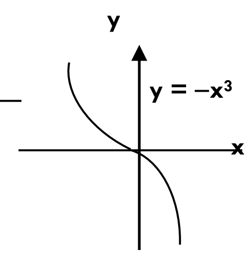
$y = f(-x)$

نستبدل كل x بـ − x









Example

If $y = x^2 - 1$
s reflected across x − axis
The new function is
$y = -(x^2 - 1)$
$y = -x^2 + 1$

Example

If $y = x^3 + 2x^2$
s reflected across y − axis
The new function is
$y = (-x)^3 + 2x^2$
$y = -x^3 + 2x^2$

ملحوظات هامة :- اذا كان مجال الدالة f(x) هو [a,b] والمدى هو [c,d] و k عدد صحيح فان

F(x)	Domain	range
F(x)	[a,b]	[c,d]
F(x+k)	[a-k,b-k]	[c,d]
F(x-k)	[a+k,b+k]	[c,d]
F(kx)	$[\frac{a}{k},\frac{b}{k}]$	[c,d]
$F(\frac{x}{k})$	[ka,kb]	[c,d]
F(x) + k	[a,b]	[c+k,d+k]
F(x) - k	[a,b]	[c-k,d-k]
kF(x)	[a,b]	[kc,kd]
$\frac{f(x)}{k}$	[a,b]	$[\frac{c}{k},\frac{d}{k}]$