• 1.5 Rational Expressions
الصيغ والمقادير الكسرية

- Rational Expression
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- Multiplication and Division
- Addition and Subtraction
- Complex Fractions.

Rational Expression

The quotient of two polynomials p and Q ,

with $Q \neq 0$,is a rational expression :

$$\frac{x+6}{x+2}, \qquad \frac{(x+6)(x+4)}{(x+2)(x+4)}, \qquad \frac{2p^2+7p-4}{5p^2+20p}$$

The domain

The domain of a rational expression is the set of real numbers for which the expression is defined.

Example 1 : Finding the domain.

Find the domain of the rational expression

$$a)\ \frac{x+6}{x+2},$$

Find the domain of the rational expression

$$b)\ \frac{(x+6)(x+4)}{(x+2)(x+4)}$$

Find the domain

$$\frac{x^2-16}{x^2-4x-12}$$

A. $\{x|x \neq -6,2\}$

B. $\{x|x \neq -4,4\}$

C. $\{x|x \neq -4,3\}$

D. $\{x|x \neq -2,6\}$

Choose the correct domain for this rational expression

$$\frac{20x + 90}{70}$$

A. $\{x|x \neq -70\}$

B. $\{x|x \neq 20\}$

C. $\{x|x \neq 90\}$

D. (All real numbers)

Lowest Terms of Rational Expression

A rational expression $\frac{a}{b}$ is written in lowest terms when the greatest common factor of its numerator a and denominator b is 1.

Examples:

$\frac{2}{3}, \frac{3}{5}, \frac{7}{8}, \ldots$ are in lowest terms

$\frac{2}{4}, \frac{5}{10}, \frac{3}{15}, \ldots \ldots$ are not in lowest terms

Fundamental Principle of Fractions: المبدأ الاساسى

$$\frac{ac}{bc} = \frac{a}{b} \quad (b \neq 0, c \neq 0), \qquad \left(\frac{a+c}{b+c} \neq \frac{a}{b}\right)$$

Examples:

$$\frac{14}{21} = \frac{2 \cdot 7}{3 \cdot 7} = \frac{2}{3}, \qquad \frac{25}{15} = \frac{5 \cdot 5}{3 \cdot 5} = \frac{5}{3},$$

$$\frac{5}{8} = \frac{2+3}{5+3} \neq \frac{2}{5}$$

Homework 1:

Write each rational expression in lowest terms:

$$a)\ \frac{2x^2+7x-4}{5x^2+20x}$$

$$b)\ \frac{6-3x}{x^2-4}$$

Simplify this rational expression to its lowest terms

$$\frac{1-w}{w^2-1}$$

A. $-(w+1)$

B. $(w+1)$

C. $-\frac{1}{w+1}$

D. $\frac{1}{w+1}$

Simplify this rational expression to its lowest terms

$$\frac{9x^4 - 27x^6}{3x^3}$$

A. $3x(1 - 3x)$

B. $3x(1 - 9x^5)$

C. $3x(1 - 3x^2)$

D. $9x^3(1 - x)$

Multiplication and Division

Multiplication and Division ضرب وقسمة الكسور

For fractions $\frac{a}{b}, \frac{c}{d}$ $(b \neq 0, d \neq 0)$, the following hold.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{and} \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad (c \neq 0)$$

Examples: $\frac{2}{7} \cdot \frac{3}{5} = \frac{2 \cdot 3}{7 \cdot 5} = \frac{6}{35}$, $\frac{4}{3} \div \frac{5}{7} = \frac{4}{3} \cdot \frac{7}{5} = \frac{4 \cdot 7}{3 \cdot 5} = \frac{28}{15}$

Example 2: Multiplying or Dividing Rational Expression s

Multiply or divided , as indicated

$a)\ \dfrac{2y^2}{9} \cdot \dfrac{27}{8y^5}$

$$b)\ \frac{3m^2-2m-8}{3m^2+14m+8}\cdot\frac{3m+2}{3m+4}$$

c) $\frac{3p^2+11p-4}{24p^3-8p^2} \div \frac{9p+36}{24p^4-36p^3}$

$$d)\ \frac{x^3 - y^3}{x^2 - y^2} \cdot \frac{2x + 2y + xz + yz}{2x^2 + 2y^2 + zx^2 + zy^2}$$

•ADDITION AND SUBTRACTION

Addition and Subtraction

For fractions $\frac{a}{b}, \frac{c}{d}$ $(b \neq 0, d \neq 0)$, the following hold.

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \quad \text{and} \quad \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd} \quad \text{and} \quad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

Examples: Find (the Least Common Dominator LCD)

$$\frac{2}{5}+\frac{3}{5}=\frac{2+3}{5}=\frac{5}{5}=1,$$

$$\frac{2}{5}-\frac{3}{5}=\frac{2-3}{5}=\frac{-1}{5}$$

$$\frac{2}{7}+\frac{3}{5}=\frac{2\cdot 5+7\cdot 3}{7\cdot 5}=\frac{31}{35}$$

$$\frac{4}{3}-\frac{5}{7}=\frac{4\cdot 7-3\cdot 5}{3\cdot 7}=\frac{28-15}{21}=\frac{13}{21}$$

•ADDITION AND SUBTRACTION

Homework 2: Addition and Subtraction

Add or subtract, as indicated

a) $\frac{5}{9x^2} + \frac{1}{6x}$

b) $\frac{y}{y-2} + \frac{8}{2-y}$

c) $\frac{3}{(x-1)(x+2)} - \frac{1}{(x+3)(x-4)}$

Indicate if the following is True or False

The least common denominator (LCD) of $\frac{1}{6a^2}$ and $\frac{1}{4ab^3}$ is $2ab$

A. True

B. False

Indicate if the following is True or False

The least common denominator (LCD) of $\frac{1}{x-2}$ and $\frac{1}{x+2}$ is $x^2 - 4$

A. True

B. False

•Complex Fractions

Example 3: Simplifying Complex Fractions

Simplify each complex fraction.

a) $\dfrac{6-\frac{5}{k}}{1+\frac{5}{k}}$

Example 3: Simplifying Complex Fractions

Simplify each complex fraction.

$$b)\ \frac{\frac{a}{a+1}+\frac{1}{a}}{\frac{1}{a}+\frac{1}{a+1}}$$