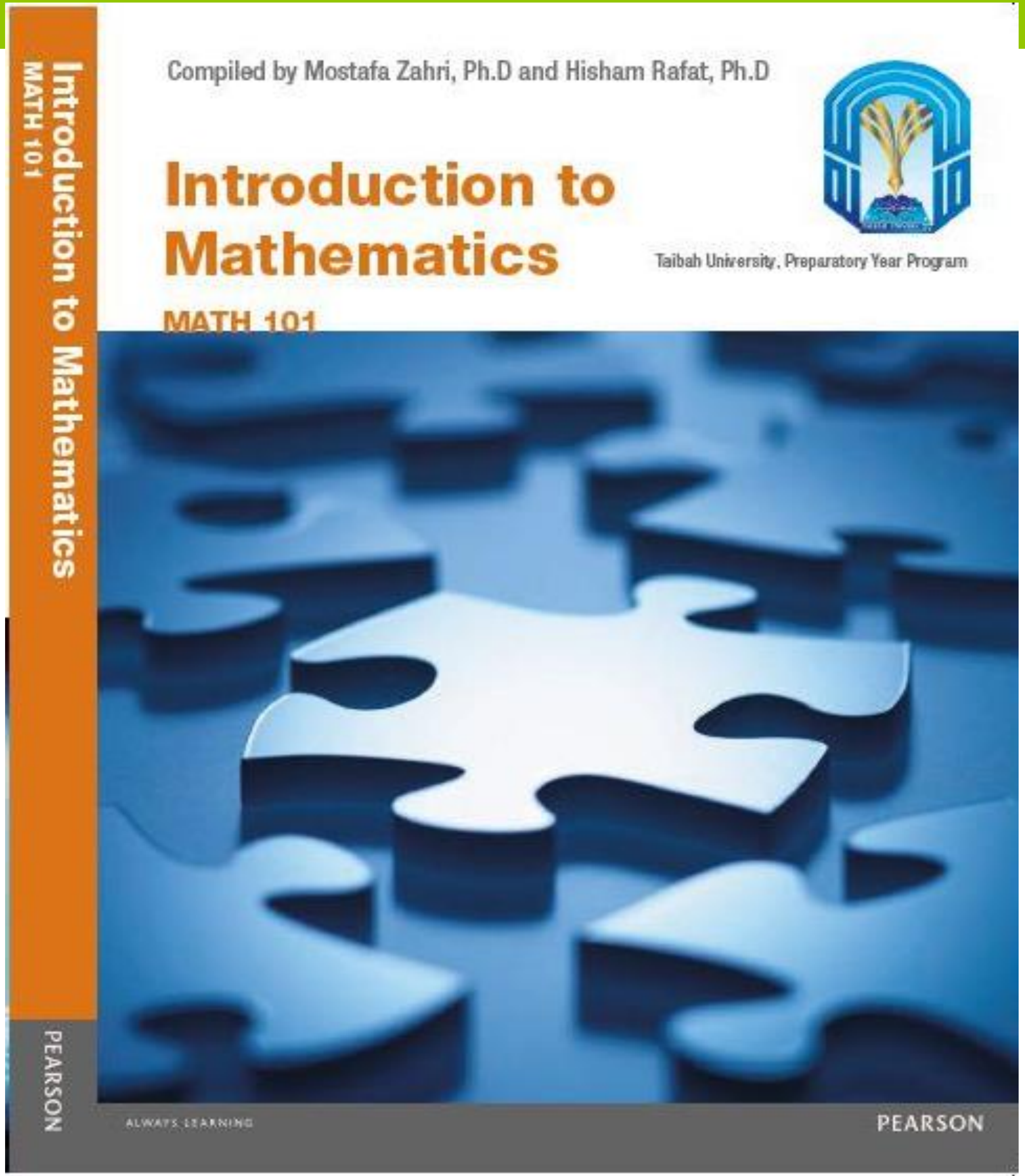
- Math-101

- Chaaper-1.2



Real Numbers and their Properties

1.2

- Sets of numbers and the Number Line
- Exponents
- Order of Operations
- Properties of Real Numbers
- Order on the Number Line
- Absolute Value

1.2 Real Numbers and Their Properties

Sets of Numbers and the Number line.

1-Natural number N={1,2,3,…}

2-Whole numbers W={0,1,2,3,…}

3- Integers I={…,-3,-2,-1,0,1,2,3,…}

4- Rational numbers

Q={$\frac{p}{q}$ |p and q are $integers$ and $q \neq 0$}

$N \subseteq W \subseteq I \subseteq Q$

1.2 Real Numbers and Their Properties

Rational numbers contains:

$2=\frac{2}{1}=\frac{4}{2}, \ldots, \quad \sqrt{4}=2, \sqrt{9}=3, \ldots \ldots$

$0=\frac{0}{1}=\frac{0}{2}, \ldots$

$\frac{1}{2}, \frac{2}{3}, \ldots .$

$0.75=\frac{3}{4}, 0.758=\frac{758}{100}, \ldots$

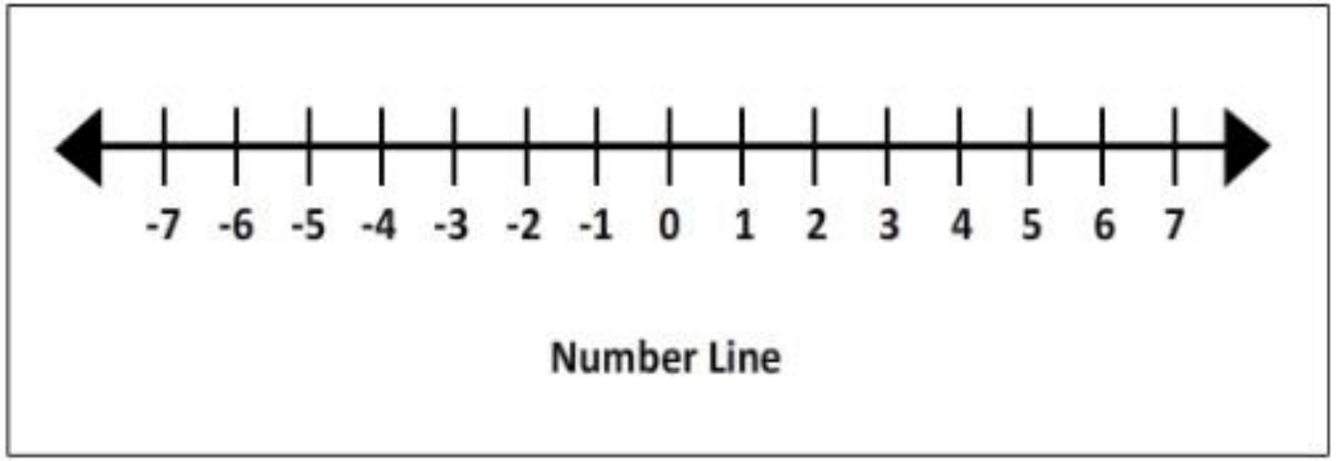
$0.\overline{6}=0.6666\ldots$

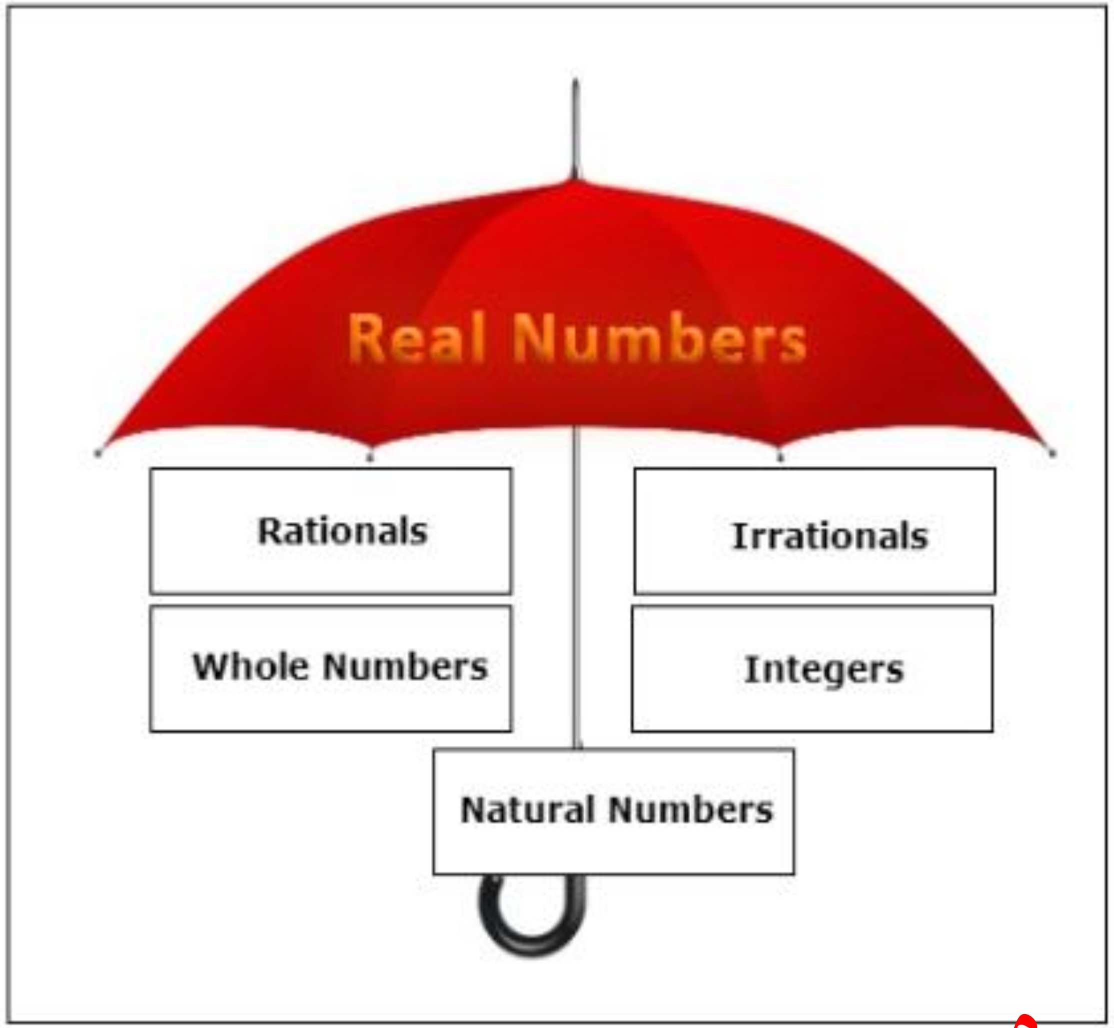
1.2 Real Numbers and Their Properties

Irrational numbers contains: Q

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 0.4785 \ldots \ldots$

Real Numbers =R





Set	Description
Natural numbers	{1, 2, 3, 4, ...}
Whole numbers	{0, 1, 2, 3, 4, ...}
Integers	{...,-3, -2, -1, 0, 1, 2, 3, ...}
Rational numbers	$\{\frac{p}{q} \mid p$ and q are integers and $q \neq 0\}$
Irrational numbers	$\{x \mid x$ is real but not rational$\}$
Real numbers	$\{x \mid x$ corresponds to a point on a number line$\}$

Example 1

Let $A = \left\{-8, -6, -\frac{12}{4}, -\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1, \sqrt{2}, \sqrt{5}, 6\right\}$.

List the elements from A that belong to each set.

a) Natural numbers

b) b) Whole numbers

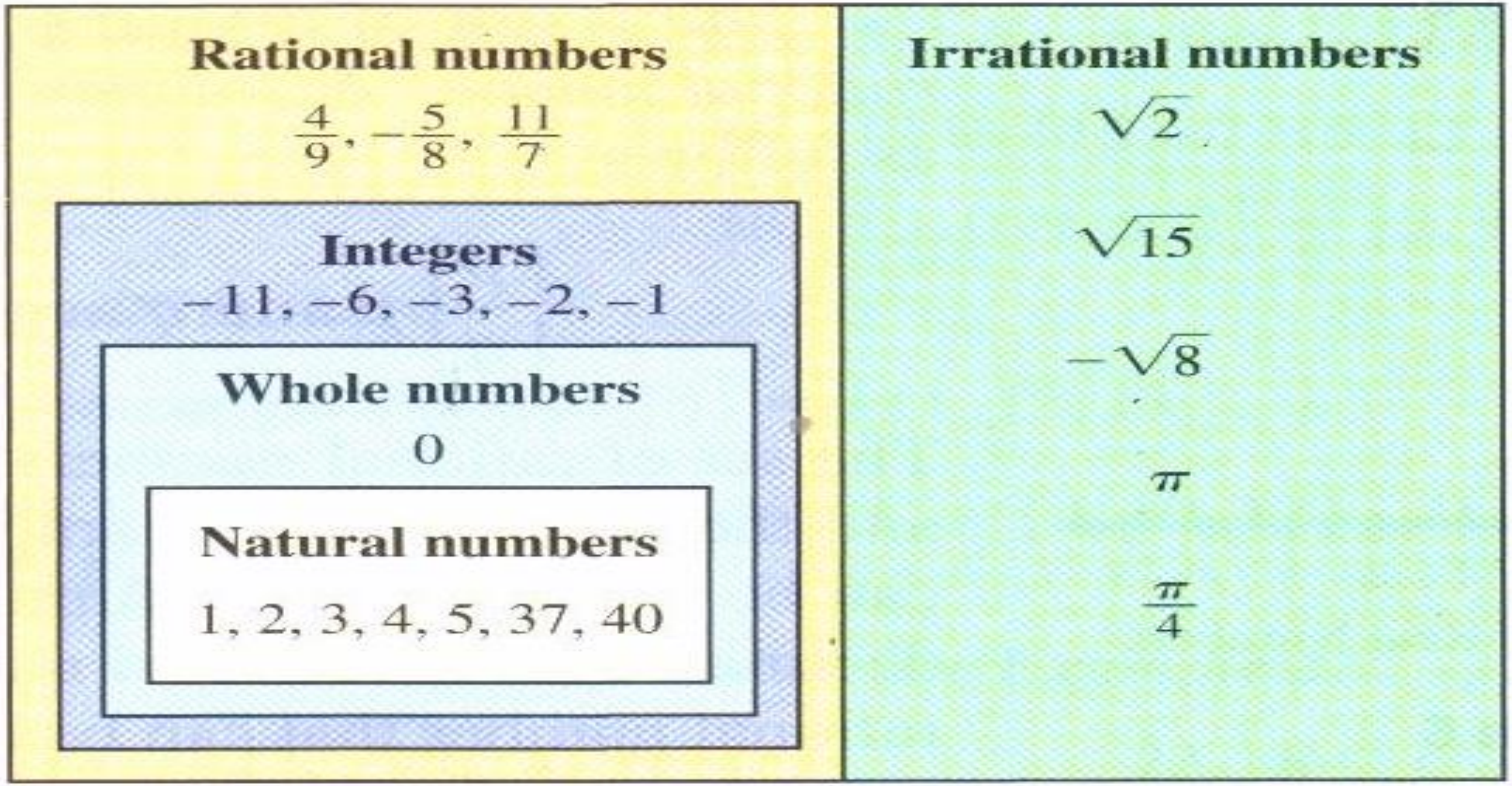
c) c) Integers

d) Rational numbers

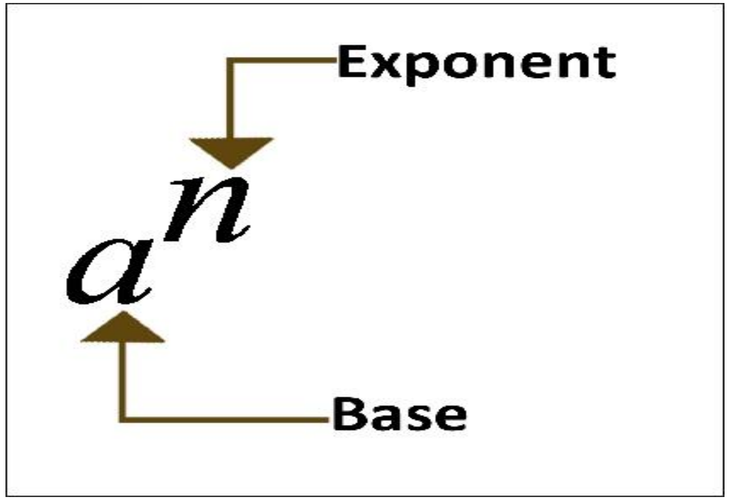
e) Irrational numbers

f) Real numbers

The real numbers



Exponents



- **Exponents**
- **What are 'Exponents'?**
- Exponents :

$$\mathbf{a^n = a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}$$

n factors of a

This is known as exponential notation. Put simply, $\mathbf{a^n}$ means **a** is multiplied by itself ***n*** times. In math, we say $\mathbf{a^n}$ is **a** to the $\mathbf{n^{th}}$ power.

- In the expression a^n,
- **a** is known as the **base**, and ***n*** is known as the **exponent**

Lows of exponents

1- $a^x \cdot a^y = a^{x+y}$

2- $\frac{a^x}{a^y} = a^{x-y}$

3- $(a^x)^y \rightarrow = a^{x \cdot y}$

4- $(a \cdot b)^x \rightarrow = a^x \cdot b^x$

$5 - (\frac{a}{b})^x = \frac{a^x}{b^x}$

Homework1

Evaluate each exponential expression ,and identify the base and the exponent.

$a)\ \ 4^3$

b)$(-6)^2$

c)-6^2

d)4.3^2

e) (4.3) ^{2}

Order of operations

Rules for Order of Operations

Let's summarize the order in which we should perform operations when simplifying or evaluating expressions.

Step 1 Treat both parts of fractions separately

Work separately above and below each fraction bar

Step 2 Parentheses ( )

Use the rules that follow within each set of parentheses or square brackets first. Start with the innermost set and work outward.

Step 3 Exponents x^y

Simplify all powers. Work from left to right.

Step 4 Roots √

Simplify all roots. Work from left to right.

Step 5 Multiplications and divisions × ÷

Do any multiplications or divisions in order. Work from left to right.

Step 6 Additions and subtractions + –

Do any additions or subtractions in order. Work from left to right.

Examples

1) $8 + (3 \times 7^2 + 5)$

$= 8 + (3 \times 49 + 5)$	Work within brackets, exponents
$= 8 + (117 + 5)$	Multiplication
$= 8 + 122$	Addition within bracket
$=130$	Addition

2) $(3^2 + (18 \div 9 + 3^2)) + 5^2$

$= (3^2 + (18 \div 9 + 9)) + 5^2$	Innermost bracket, exponent
$= (3^2 + (2 + 9)) + 5^2$	Division
$= (3^2 + 11) + 5^2$	Addition
$= (9 + 11) + 5^2$	Next bracket, exponent
$x= 20 + 25$	Addition within bracket, exponent
$= 45$	Addition

Example 2:

Evaluate each expression.

$a)\ 6 \div 3 + 2^3 . 5$

$b)\ (8+6) \div 7 \cdot 3 - 6$

$c)\ \frac{4+3^2}{6-5 \cdot 3}$

d) $\frac{-(-3)^3+(-5)}{2(-8)-5(3)}$

Homework 2: Using order of Operations

Evaluate each expression for $x = -2, y = 5, and\ z = -3$

$a)\ -4x^2 - 7y + 4z$

$b)\ \frac{2(x-5)^2 + 4y}{z+4}$

Properties of Real numbers

The Commutative Property of Addition

Look at this expression:

$$\mathbf{4 + 5 = 9}$$

is the same as **5 + 4 = 9**

This is an example of **commutative property**. It means that we can move the numbers around in an addition sum and still get the same answer.

Properties of Real numbers

- The Commutative Property of Multiplication and the Closure Property
- As with addition, the commutative property works for multiplication too.

4 x 5 = 20 is the same as5 x 4 = 20

.

- Another property of real numbers,

the closure property, states that those answers will be real numbers.

- The sum of two real numbers is a real number –the additive closure property.
- The product of two real numbers is a real number –the multiplicative closure property.

The Associative Property of Addition

Look at this expression:

3x + (4x + 6)is the same as **(3x + 4x) + 6**

This is an example of **associative property**.

The Associative Property of Multiplication

As with addition, the associative property works for multiplication too.

3 (2x)is the same as**(3 ×2)x**

The **associative property** of real numbers allows us to regroup numbers in additions and multiplications, making such operations simpler.

The commutative and associative properties we have described only apply to addition and multiplication. They do not work with subtraction or division.

This means, for example, that for any two numbers, a and b, where $a \neq b$:

$$a - b \neq b - a \quad \text{and} \quad a \div b \neq b \div a$$

~~Subtraction~~ ~~Division~~

Additive Identity

Now let's consider the identity property. The identity property for addition is a number that when added to any number does not change the value of that number.

The additive identity for real numbers is 0. This means that adding 0 to any number doesn't change that number's value.

Example:

3+0=3

458+0=458

In general terms, there exists a unique real number 0 such that

This is known as the additive identity

Multiplicative Identity

Is there an identity property for multiplications too?
Yes, there is multiplicative identity. It means that when we multiply 1 by any number we get the same number, which means that it keeps its identity.

Example:

$$8 \times 1 = 8$$

$$254 \times 1 = 254$$

In general terms, there exists a unique real number 1 such that
This is known as the multiplicative identity

Property	Equations
Commutative property	$a + b = b + a$ $ab = ba$
Closure property	$a + b$ is a real number ab is a real number
Associative property	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$
Identity property	$a + 0 = a$ $a \cdot 1 = a$
Inverse property	$a + (-a) = 0$ $a \cdot \frac{1}{a} = 1$ for $a \neq 0$
Distributive property	$a(b + c) = ab + bc$

Example 3

Use the commutative and associative properties to simplify each expression

$a)\ 6 + (9 + x)$

$b)\ \frac{5}{8}(16y)$

$c)\ -10p(\frac{6}{5})$

Homework 3

Rewrite each expression using the distributive property and simplify, if possible.

$a)\ 3(x + y)$

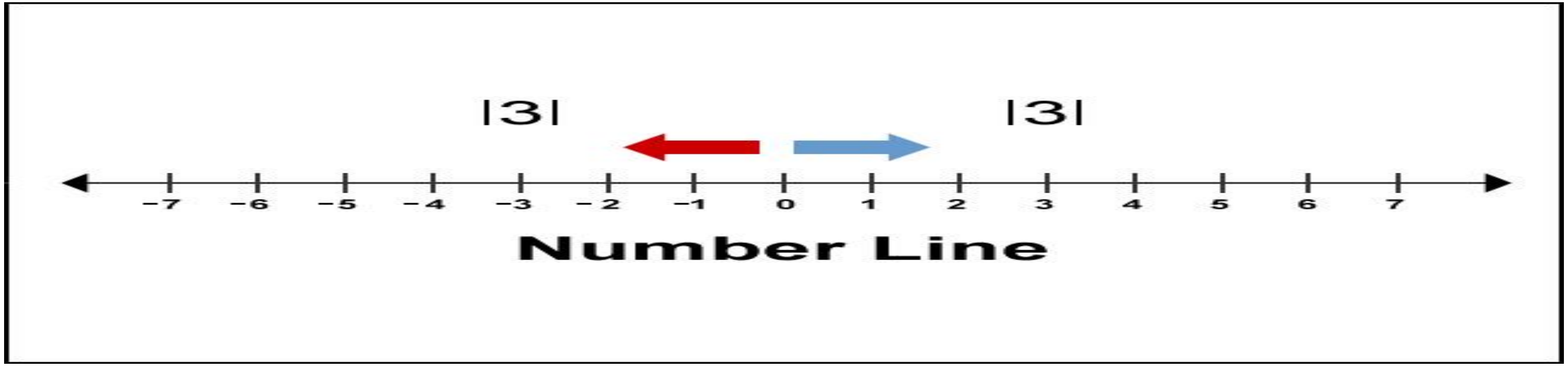
$b)\ -(m - 4n)$

$$c)\ \frac{1}{3}\left(\frac{4}{5}m-\frac{3}{2}n-27\right)=$$

$$d)\ 7p+21=$$

Absolute Value

Absolute value is the distance of any number from 0 on a number line in any direction. The direction doesn't change the distance; it is always positive. So whether we are finding absolute value for negative or positive numbers, the absolute value is always positive.



Examples:

$$|-3| =$$

$$-|8| =$$

$$|3| =$$

$$-|-2| =$$

$$|-5| =$$

$$\left|-\frac{3}{8}\right| =$$

Homework4:

Let x=-6, and y=10. Evaluate each expression :

:

a) $|2x - 3y| =$

$$\frac{2|x| - |3y|}{|xy|} =$$

Distance between points on a number line

If P and Q are two points on a number line with coordinates a and b, respectively, then the distance d(P,Q) between them is given by the following

$d(P,Q)=|a-b|=$ or $d(P,Q)=|b-a|$

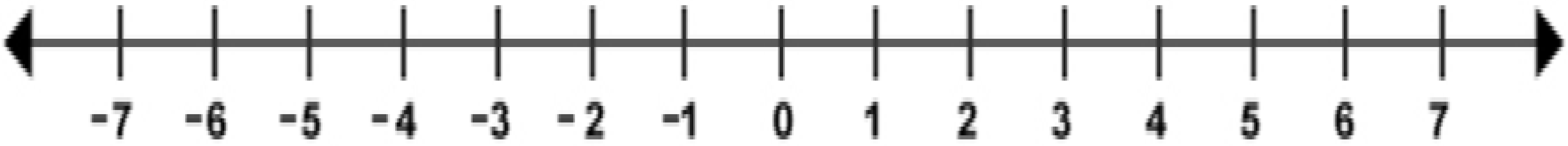
Example 5:

Find the distance between -5 and 8.

Distance between points on a plane

if : P $(-1 , 2)$ and Q $(3 , 4)$ The distance between P and Q. is if : P $(-1 , 0)$ and Q $(0 , 4)$ The distance between P and Q. is if : P (-9, 0) and Q (-10 , 4) The distance between P and Q. is

- Order on the Number Line



If the real number a is to the left of the real number b on a number line, then

a is less than b, written $a < b$

If a is to the right of b, then

a is greater than b, written $a > b$

Also we have

$a \leq b$ (a is less than or equal b)

$a \geq b$ (a is greater than or equal b)

$a < b < c$, b is between a and c.