• 2.1

- Basic Terminology of Equations
- Solving Linear Equations
- Identities, Conditional Equations, and Contradictions
- Solving for a Specified Variable (Literal Equations)

Basic Terminology of Equations

An equation is a statement that two expressions are equal.

$x + 2 = 9$ $\quad 11x = 5x + 6x$ $\quad x^2 - 2x - 1 = 0$

To solve an equation means to find all numbers that make the equation a true statement. These numbers are the **solutions,** or **roots,** of the equation. A number that is a solution of an equation is said to *satisfy* the equation, and the solutions of an equation make up its **solution set.** Equations with the same solution set are **equivalent equations.**

Addition and Multiplication Properties of Equality

Let a, b, and c represent real numbers.

If $a = b$, then $a + c = b + c$.

That is, the same number may be added to each side of an equation without changing the solution set.

Addition and Multiplication Properties of Equality

Let a, b, and c represent real numbers.

If $a = b$ and $c \neq 0$, then $ac = bc$.

That is, each side of an equation may be multiplied by the same nonzero number without changing the solution set.

Linear Equation in One Variable

A **linear equation in one variable** is an equation that can be written in the form

$$ax + b = 0,$$

where a and b are real numbers with $a \neq 0$.

Linear Equations

A linear equation is also called a **first-degree equation** since the greatest degree of the variable is 1.

Linear equations

$$3x + \sqrt{2} = 0 \qquad \frac{3}{4}x = 12 \qquad 0.5(x + 3) = 2x - 6$$

Nonlinear equations

$$\sqrt{x} + 2 = 5 \qquad \frac{1}{x} = -8 \qquad x^2 + 3x + 0.2 = 0$$

Example 1 SOLVING A LINEAR EQUATION

Solve $3(2x-4)=7-(x+5)$.

Solution

$$3(2x-4)=7-(x+5)$$

$$6x-12=7-x-5 \quad \text{Distributive property}$$

$$6x-12=2-x \quad \text{Combine like terms.}$$

$$6x-12+x=2-x+x \quad \text{Add } x \text{ to each side.}$$

$$7x-12=2 \quad \text{Combine like terms.}$$

$$7x-12+12=2+12 \quad \text{Add 12 to each side.}$$

$$7x=14 \quad \text{Combine like terms.}$$

$$\frac{7x}{7}=\frac{14}{7},\ x=2 \quad \text{Divide each side by 7.}$$

Example 2 SOLVING A LINEAR EQUATION WITH FRACTIONS

Solve $\dfrac{2x+4}{3}+\dfrac{1}{2}x=\dfrac{1}{4}x-\dfrac{7}{3}.$

Identities, Conditional Equations, and Contradictions

An equation satisfied by every number that is a meaningful replacement for the variable is an **identity.**

$$3(x+1)=3x+3$$

An equation that is satisfied by some numbers but not others is a **conditional equation.**

$$2x=4$$

An equation that has no solution is a **contradiction**.

$$x=x+1$$

Example 3 IDENTIFYING TYPES OF EQUATIONS

Determine whether each equation is an *identity*, a *conditional equation*, or a *contradiction*.

(a) $-2(x+4)+3x=x-8$

Example 3 IDENTIFYING TYPES OF EQUATIONS

Determine whether each equation is an *identity*, a *conditional equation*, or a *contradiction*.

(b) $5x - 4 = 11$

Example 3 IDENTIFYING TYPES OF EQUATIONS

Determine whether each equation is an *identity*, a *conditional equation*, or a *contradiction*.

(c) $3(3x-1)=9x+7$

Identifying Types of Linear Equations

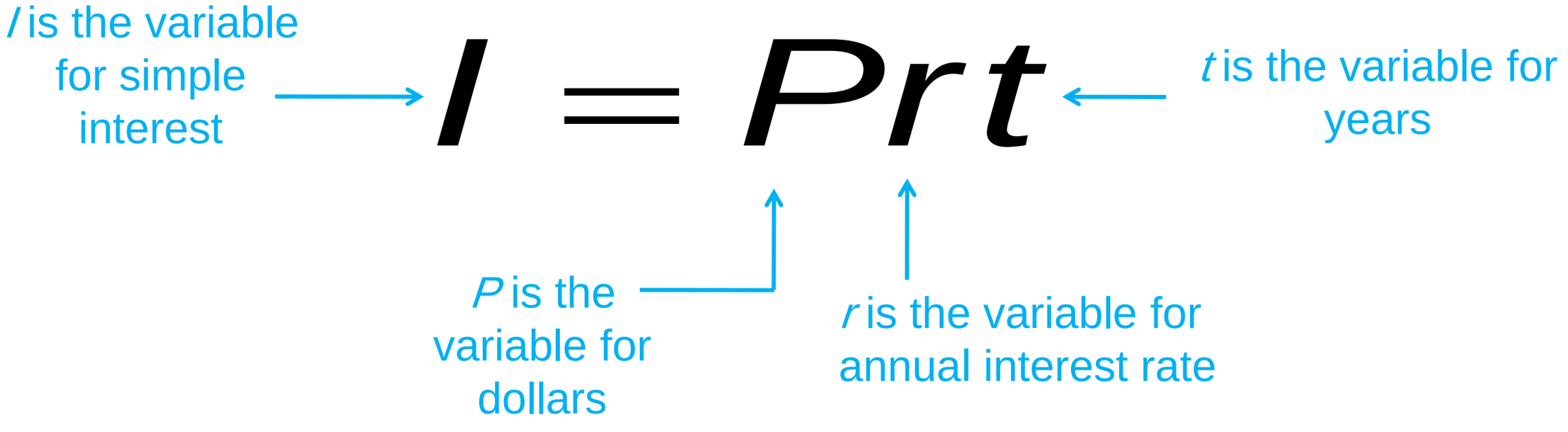
1-If solving a linear equation leads to a true statement such as $0 = 0$,
the equation is an **identity**. Its solution set is **{all real numbers}.**

2-If solving a linear equation leads to a single solution such as $x = 3$,
the equation is **conditional**. Its solution set consists of a single element.

3-If solving a linear equation leads to a false statement such as $-3 = 7$,
then the equation is a **contradiction**. Its solution set is Φ .

Solving for a Specified Variable (Literal Equations)

A formula is an example of a **linear equation** (an equation involving letters). This is the formula for **simple interest.**



Solving for a Specified Variable (Literal Equations)

This formula gives the **future value,** or **maturity value,** A of P dollars invested for t years at an annual simple interest rate r.

A is the variable for future or maturity value

$$A = P(1 + r\,t)$$

t is the variable for years

P is the variable for dollars

r is the variable for annual simple interest rate

SOLVING FOR A SPECIFIED VARIABLE

(a) Solve for *t*. $I = Prt$

(b) Solve for *P*. $A - P = Prt$

(c) Solve for *x*. $3(2x - 5a) + 4b = 4x - 2$