❖ 2.2- Complex Numbers

- Basic Concepts of Complex Numbers
- Operations on Complex Numbers

Basic Concepts of Complex Numbers

$$x^2 = -1.$$

1- since no real number, when squared, gives $-1.$

To extend the real number system to include solutions of equations of this type, the number *i* is defined to have the following property.

$$i = \sqrt{-1}, \text{ and therefore, } i^2 = -1.$$

2- If a and b are real numbers, then any number of the form $\boldsymbol{a + bi}$ is a **complex number.**

3- In the complex number $a + bi$, a is the **real part** and b is the **imaginary part**.

4-Two complex numbers $a + bi$ and $c + di$ are equal provided that their real parts are equal and their imaginary parts are equal; that is

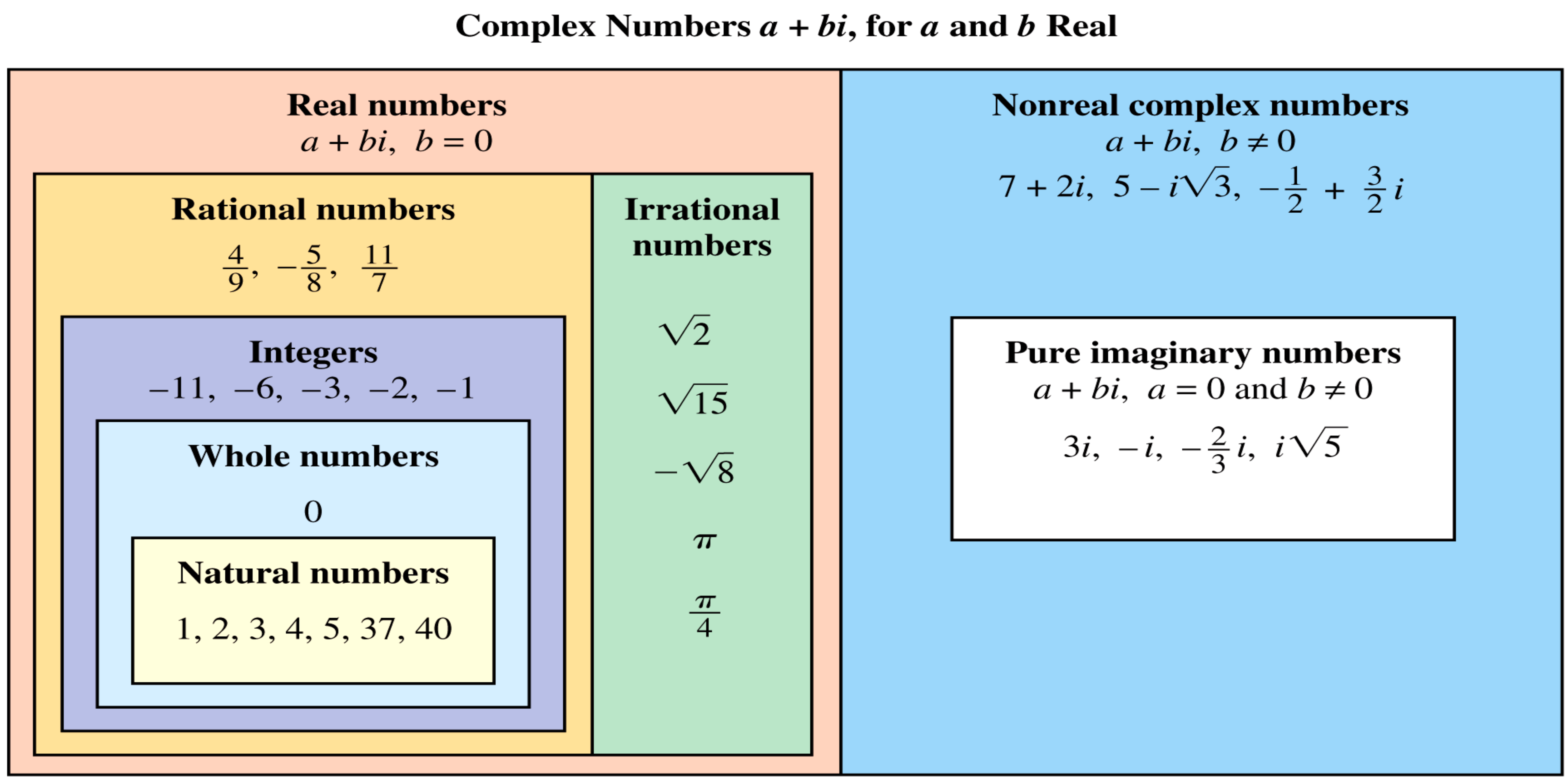
$a + bi = c + di$ if and only if $a = c$ and $b = d$.

5- For complex number $a + bi$, if $b = 0$, then $a + bi = a$.
Thus, the set of real numbers is a subset of the set of complex numbers.

6- If $a = 0$ and $b \neq 0$, the complex number is said to be a **pure imaginary number.**

7-A pure imaginary number, or a number like $7 + 2i$ with $a \neq 0$ and $b \neq 0$, is a **nonreal complex number.**

8-- A complex number written in the form $a + bi$ (or $a + ib$) is in **standard form.**


Complex Numbers $a + bi$, for a and b Real
Real numbers
$a + bi,\ b = 0$
Rational numbers
$\frac{4}{9},\ -\frac{5}{8},\ \frac{11}{7}$
Integers
$-11,\ -6,\ -3,\ -2,\ -1$
Whole numbers
0
Natural numbers
1, 2, 3, 4, 5, 37, 40
Irrational numbers
$\sqrt{2}$
$\sqrt{15}$
$-\sqrt{8}$
π
$\frac{\pi}{4}$
Nonreal complex numbers
$a + bi,\ b \neq 0$
$7 + 2i,\ 5 - i\sqrt{3},\ -\frac{1}{2} + \frac{3}{2}i$
Pure imaginary numbers
$a + bi,\ a = 0$ and $b \neq 0$
$3i,\ -i,\ -\frac{2}{3}i,\ i\sqrt{5}$

THE EXPRESSION $\sqrt{-a}$

If $a > 0$, then $\sqrt{-a} = i\sqrt{a}$.

Example 1

Write as the product of a real number and *i*, using the definition of $\sqrt{-a}$.

(a) $\sqrt{-16}$

(b) $\sqrt{-70}$

(c) $\sqrt{-48}$

Simplify using real numbers and i.

$$\sqrt{-9}$$

A. ± 3

B. $-i\sqrt{3}$

C. $-3i$

D. $3i$

Simplify using real numbers and i.

$$\sqrt{-49}$$

A. $-7i$

B. $7i$

C. ± 7

D. $\sqrt{7}$

Simplify using real numbers and *i*.

$$2\sqrt{-72}$$

A. $2\sqrt{2}$

B. $-12\sqrt{2}$

C. $12i\sqrt{2}$

D. $6i\sqrt{8}$

Operations on Complex Numbers

Products or quotients of Complex Numbers

Products or quotients with negative radicands are simplified by first rewriting

$$\sqrt{-a} \text{ as } i\sqrt{a}$$

for a positive number *a*.

Then the properties of real numbers and the fact that are applied

$$i^2 = -1$$

Caution ***When working with negative radicands, use the definition***

$$\sqrt{-a} = i\sqrt{a}$$

before using any of the other rules for radicals.

FINDING PRODUCTS AND QUOTIENTS INVOLVING $\sqrt{-a}$

Homework 1 **Multiply or divide, as indicated. Simplify each answer.**

(a) $\sqrt{-7} \times \sqrt{-7}$

(b) $\sqrt{-6} \times \sqrt{-10}$

(c) $\dfrac{\sqrt{-20}}{\sqrt{-2}}$

(d) $\dfrac{\sqrt{-48}}{\sqrt{24}}$

Example 2

Write $\dfrac{-8+\sqrt{-128}}{4}$ in standard form $a + bi$.

Addition and Subtraction of Complex Numbers

For complex numbers $a + bi$ and $c + di$,

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

and $(a+bi)-(c+di)=(a-c)+(b-d)i.$

Homework 2

Find each sum or difference.

(a) $(3-4i)+(-2+6i)$

(b) $(-4+3i)-(6-7i)$

Multiplication of Complex Numbers

The product of two complex numbers is found by multiplying as though the numbers were binomials and using the fact that $i^2 = -1$, as follows.

For complex numbers $a + bi$ and $c + di$,

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i.$$

Example 3 Find each product.

(a) $(2-3i)(3+4i)$

(b) $(4+3i)^2$

(c) $(6+5i)(6-5i)$

Property of Complex Conjugates

For real numbers *a* and *b*,

$$(a+bi)(a-bi)=a^2+b^2.$$

DIVIDING COMPLEX NUMBERS

Homework 3

Write each quotient in standard form $a + bi$.

(a) $\dfrac{3+2i}{5-i}$

(b) $\dfrac{3}{i}$

Powers of i

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$ and so on.

Simplifying Powers of *i*

Powers of *i* can be simplified using the facts

$$i^2 = -1 \quad \text{and} \quad i^4 = (i^2)^2 = (-1)^2 = 1.$$

$$i^n \rightarrow \frac{n}{4} = \begin{cases} m(integer) \rightarrow i^n = 1 \\ m.25 \rightarrow i^n = i \\ m.50 \rightarrow i^n = -1 \\ m.57 \rightarrow i^n = -i \end{cases}$$

Simplifying Powers of *i*

Powers of *i* can be simplified using the facts

$$i^2 = -1 \quad \text{and} \quad i^4 = (i^2)^2 = (-1)^2 = 1.$$

$$i^n \rightarrow \frac{n}{4} = \begin{cases} m(integer) i^{4m} = 1 \\ i^{4m+1} = i \\ i^{4m+2} = -1 \\ i^{4m+3} = -i \end{cases}$$

Simplifying Powers of *i*

Powers of *i* can be simplified using the facts

$$i^2 = -1 \quad \text{and} \quad i^4 = (i^2)^2 = (-1)^2 = 1.$$

$$i^{-n} \to \frac{n}{4} = \begin{cases} m(integer) \to i^{-n} = 1 \\ m.25 \to i^{-n} = -i \\ m.50 \to i^{-n} = -1 \\ m.57 \to i^{-n} = i \end{cases}$$

Example 4

Simplify each power of *i*.

(a) i^{15}

(b) i^{-3}

Simplify and write in the standard form of a complex number $\boldsymbol{i}^{16}$

A. $\mathbf{1}$

B. $-i$

C. $-\mathbf{1}$

D. $\boldsymbol{i}$

x

Simplify and write in the standard form of a complex number i^{21}

A. 1

B. $-i$

C. -1

D. i

Simplify and write in the standard form of a complex number i^{42}

A. 1

B. $-i$

C. -1

D. i

Simplify and write in the standard form of a complex number $\frac{2}{5-3i}$

A. $\frac{5}{8}-\frac{3}{8}i$

B. $\frac{5}{8}+\frac{3}{8}i$

C. $\frac{5}{17}+\frac{3}{17}i$

D. $\frac{5}{17}-\frac{3}{17}i$