2.4 Inequalities

- Linear Inequalities
- Three-Part Inequalities
- Quadratic Inequalities
- Rational Inequalities

Properties of Inequality

Let a, b and c represent real numbers.

1. If $a < b$, then $a \pm c < b \pm c$.

2. If $a < b$ and if $c > 0$, then $ac < bc$.

3. If $a < b$ and if $c < 0$, then $ac > bc$.

4.If $a < b$ and if $c > 0$, then $a/c < b/c$.

5. If $a < b$ and if $c < 0$, then $a/c > b/c$.

Example:

$2<5$ then $2+3<5+3$ i.e. $5<8$

- $2<5$, then $2-1<5-1$, i.e. $1<4$
- $2<5$, $c=3>0$, $2.3<5.3$, i.e. $6<15$
- $2<5$, $c=-3<0$, $2.-3>5.-3$, i.e. $-6>-15$
- $4<6$, $c=2>0$, $4/2<6/2$, i.e. $2<3$
- $4<6$, $c=-2<0$, $4/-2>6/-2$, i.e. $-2>-3$

Motion Problems

Note Multiplication may be replaced by division in Properties 2 and 3.

Always remember to reverse the direction of the inequality symbol when multiplying or dividing by a negative number.

Linear Inequality in One Variable

A **linear inequality in one variable** is an inequality that can be written in the form

$$ax + b > 0,$$

where a and b are real numbers, with $a \neq 0$. (Any of the symbols $\geq$, $<$, and $\leq$ may also be used.)

Example 1 SOLVING A LINEAR INEQUALITY

Solve $-3x + 5 > -7.$

Homework 1

SOLVING A LINEAR INEQUALITY

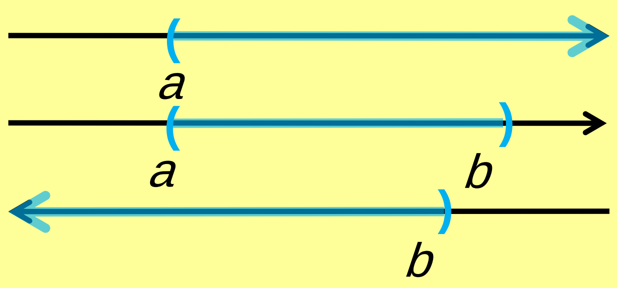
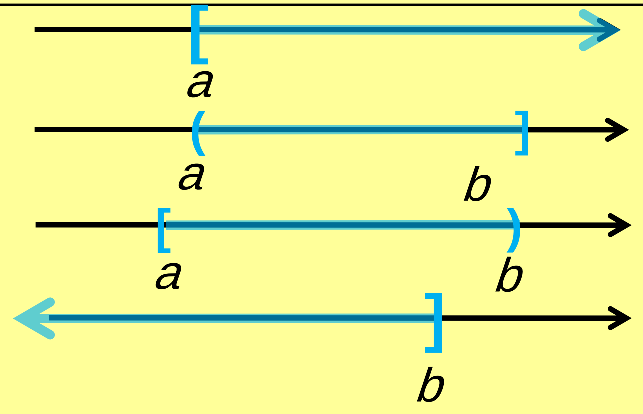
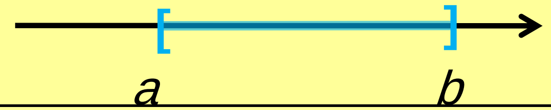
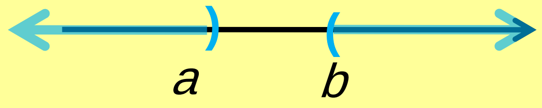
Solve $4 – 3x \leq 7 + 2x$.

Give the solution set in interval notation.

Homework 2

SOLVING A THREE-PART INEQUALITY

Solve $-2 < 5 + 3x < 20$.

Type of Interval	Set	Interval Notation	Graph
Open interval	$\{x \mid x > a\}$ $\{x \mid a < x < b\}$ $\{x \mid x < b\}$	(a, ∞) (a, b) $(-\infty, b)$	
Other intervals	$\{x \mid x \geq a\}$ $\{x \mid a < x \leq b\}$ $\{x \mid a \leq x < b\}$ $\{x \mid x \leq b\}$	$[a, \infty)$ $(a, b]$ $[a, b)$ $(-\infty, b]$	
Closed interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	
Disjoint interval	$\{x \mid x < a$ or $x > b\}$	$(-\infty, a) \cup (b, \infty)$	
All real numbers	$\{x \mid x$ is a real number$\}$	$(-\infty, \infty)$	

Quadratic Inequalities

A **quadratic inequality** is an inequality that can be written in the form

$$ax^2 + bx + c < 0$$

for real numbers *a*, *b*, and *c*, with $a \neq 0$.
(The symbol < can be replaced with >, ≤, or ≥.)

Solving a Quadratic Inequality

Step 1 Solve the corresponding quadratic equation.
Step 2 Identify the intervals determined by the solutions of the equation.
Step 3 Use a test value from each interval to determine which intervals form the solution set.

<u>Inequalities that use the symbols</u>
$<$ and $>$ are **strict inequalities**;
$\leq$ and $\geq$ are used in **nonstrict inequalities.**

11

Example 3 **SOLVING A QUADRATIC INEQULITY**

Solve $x^2 - x - 12 < 0.$

Homework 3

SOLVING A QUADRATIC INEQUALITY

Solve $2x^2 + 5x - 12 \geq 0.$

Solving a Rational Inequality

Step 1 Rewrite the inequality, if necessary, so that 0 is on one side and there is a single fraction on the other side.

Step 2 Determine the values that will cause either the numerator or the denominator of the rational expression to equal 0. These values determine the intervals of the number line to consider.

Solving a Rational Inequality

Step 3 Use a test value from each interval to determine which intervals form the solution set.

A value causing the denominator to equal zero will never be included in the solution set. If the inequality is strict, any value causing the numerator to equal zero will be excluded. If the inequality is nonstrict, any such value will be included.

Caution ***Be careful with the endpoints of the intervals when solving rational inequalities***.

Caution Solving a rational inequality such as

$$\frac{5}{x+4} \geq 1$$

by multiplying each side by $x + 4$ to obtain

$5 \geq x + 4$ requires considering *two cases*, since the sign of $x + 4$ depends on the value of x.

If $x + 4$ is negative, then the inequality symbol must be reversed.

The procedure used in the next two examples eliminates the need for considering separate cases.

Homework 4

SOLVING A RATIONAL INEQUALITY

Solve $\dfrac{5}{x+4} \geq 1.$

Example 5

SOLVING A RATIONAL INEQUALITY

Solve $$\frac{2x-1}{3x+4} < 5.$$

SOLVING A RATIONAL INEQUALITY

Solve $\dfrac{x-2}{x+4} \geq 0$