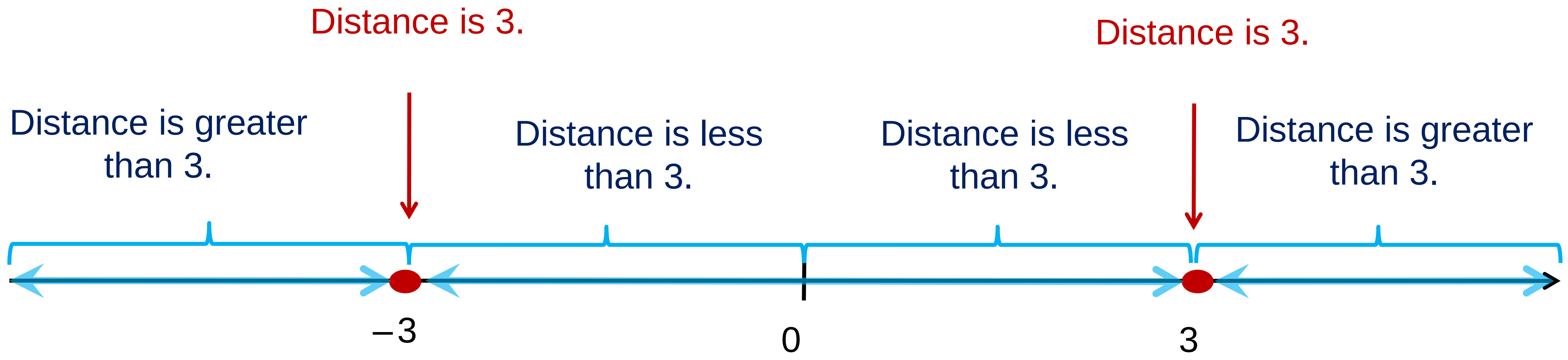
2.5 : Absolute Value Equations and Inequalities

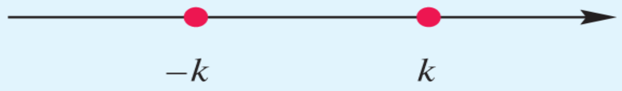
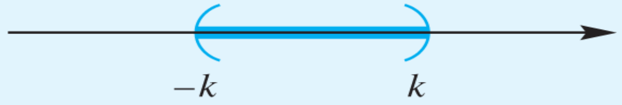
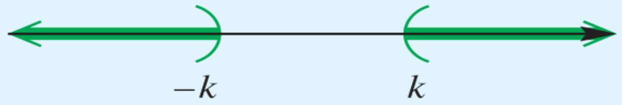
- Basic Concepts
- Absolute Value Equations
- Absolute Value Inequalities
- Special Cases
- Absolute Value Models for Distance and Tolerance

The **absolute value** of a number *a* gives the distance from *a* to 0 on a number line.



By this definition, the equation $|x| = 3$ can be solved by finding all real numbers at a distance of 3 units from 0. Two numbers satisfy this equation, 3 and –3. So the solution set is $\{-3, 3\}$.

For each equation or inequality in Cases 1-3 in the table, assume that $k > 0$.

Absolute Value Equation or Inequality	Equivalent Form	Graph of the Solution Set	Solution Set
Case 1: $\lvert x \rvert = k$	$x = k$ or $x = -k$		$\{-k, k\}$
Case 2: $\lvert x \rvert < k$	$-k < x < k$		$(-k, k)$
Case 3: $\lvert x \rvert > k$	$x < -k$ or $x > k$		$(-\infty, -k) \cup (k, \infty)$

In Cases 2 and 3, the strict inequality may be replaced by its nonstrict form. Additionally, if an absolute value equation takes the form $\lvert a \rvert = \lvert b \rvert$, then a and b must be equal in value or opposite in value.

Thus, the equivalent form of $\lvert a \rvert = \lvert b \rvert$ ***is*** $a = b$ ***or*** $a = -b$.

Example 1

SOLVING ABSOLUTE VALUE EQUATIONS

Solve each equation.

(a) $\left|5-3x\right|=12$

Example 1

SOLVING ABSOLUTE VALUE EQUATIONS

Solve each equation.

(b) $|4x-3|=|x+6|$

Homework 1

SOLVING ABSOLUTE VALUE INEQUALITIES

Solve each inequality.

(a) $|2x+1| < 7$

Homework 1

SOLVING ABSOLUTE VALUE INEQUALITIES

Solve each inequality.

(b) $|2x + 1| > 7$

Example 2

SOLVING AN ABSOLUTE VALUE INEQUALITY

Solve $|2-7x|-1>4.$

Homework 2

SOLVING SPECIAL CASES

Solve each inequality.

(a) $|2-5x| \geq -4$

Homework 2

SOLVING SPECIAL CASES

(b) $|4x-7|<-3$

Homework 2

SOLVING SPECIAL CASES

Solve each equation or inequality.

(c) $|5x + 15| = 0$

USING ABSOLUTE INEQUALITIES TO DESCRIBE DISTANCES

Write each statement using an absolute value inequality.

(a) k is no less than 5 units from 8.

(b) n is within 0.001 unit from 6.

USING ABSOLUTE VALUE TO MODEL TOLERANCE

In quality control and other applications, we often wish to keep the difference between two quantities within some predetermined amount, called the **tolerance.**

Example 6

Suppose $y = 2x + 1$

and we want y to be within 0.01 unit of 4.

For what values of x will this be true?