

# 1 Introduction to Physics, Units and Measurements

# Introduction, Measurement, Estimating

## Contents

- Measurement and Uncertainty; Significant Figures
- Units, Standards, and the SI System
- Converting Units
- Order of Magnitude: Rapid Estimating
- Dimensions and Dimensional Analysis

The major goals of this chapter are to enable you to:

1. Use the metric system of measurement.
2. Convert measurements from one system to another.
3. Use significant digits to determine the accuracy of measurements.
4. Differentiate between accuracy and precision.
5. Solve problems with measurements and consistently express the results with the correct significant digits.
6. Use a systematic approach to solving physics problems.
7. Analyze problems using the problem-solving method.

# Measurement and Uncertainty

**NO measurement is absolutely precise OR *accurate***

*So, There is an uncertainty associated with every measurement*

When giving the result of a measurement, it is important to state the **estimated uncertainty** in measurement

## **Main sources of uncertainty (errors):**

- **Human errors:**  
(*difficulty reading results Between the smallest divisions*)
- **Limited Instrument accuracy** (*systematic error*)

# Accuracy vs. Precision

## Precision

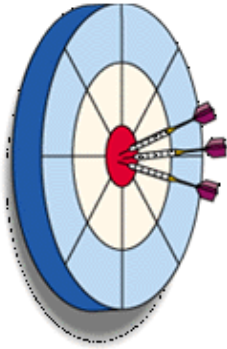
- Precision is the repeatability of the measurement using the same instrument. That is,
- Precision is how close the measured values are to each other

## Accuracy

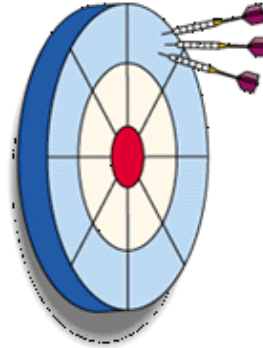
- Accuracy is how close a measurement comes to the true value.
- Accuracy is a measure of the difference between a measured value and the true value, that is, the error.
- If the errors of measurement average to zero, then the system is said to be accurate.

- **It is possible to be accurate without being precise and to be precise without being accurate!**

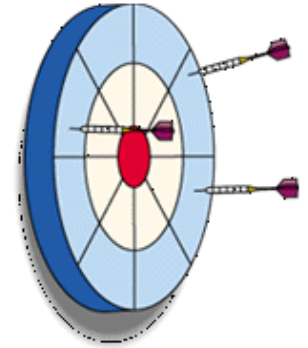
# Accuracy vs. Precision



**Good accuracy**  
**Good precision**



**Poor accuracy**  
**Good precision**



**Poor accuracy**  
**Poor precision**

Relatively accurate ruler with least reading  $\sim 0.05$  unit



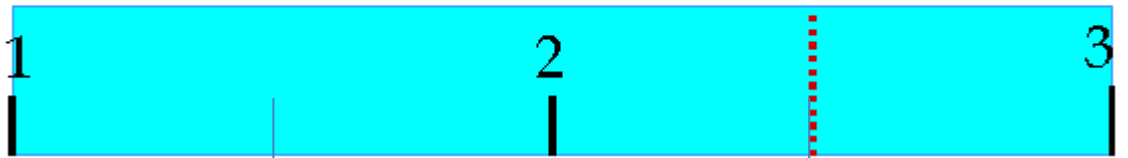
2.55



2.5

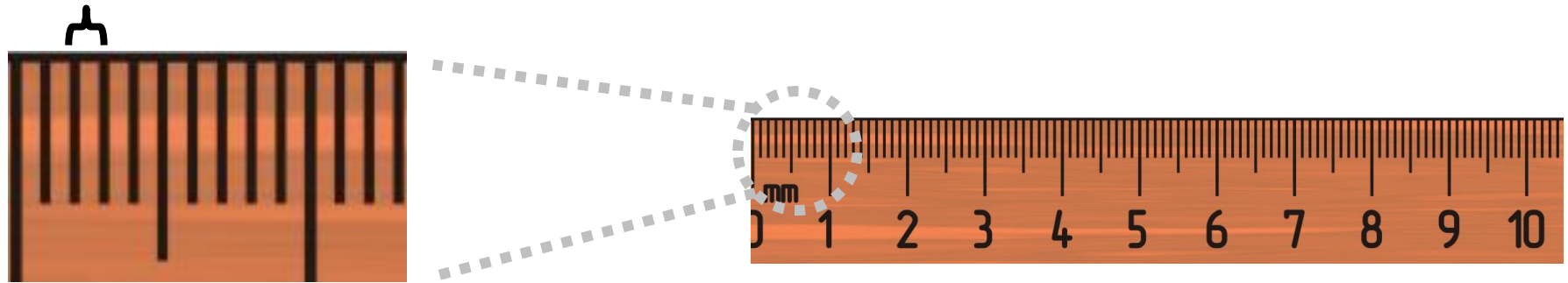


Less accurate ruler. Its least reading  $\sim 0.5$  unit



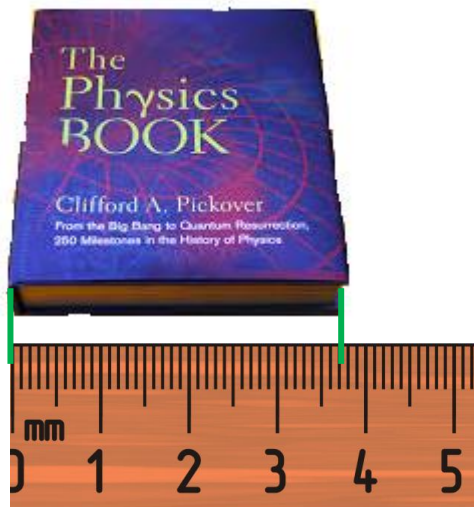
The result could be claimed to be **precise** to the **smallest** division shown

Smallest division = **1 mm = 0.1 cm**

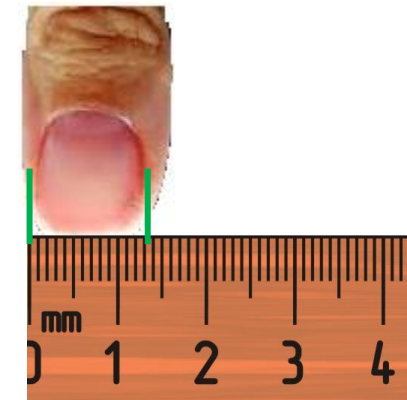


The ruler is *precise* to within 0.1 cm,

⇒ **estimated uncertainty (error) =  $\pm 0.1$  cm**



The tiny book is  **$3.7 \pm 0.1$  cm** wide  
⇒ its *true* width likely lies between **3.8 and 3.6 cm**



Width of thumbnail =  **$1.3 \pm 0.1$  cm**  
⇒ it lies between **1.4 and 1.2 cm**

**Therefore measurement result is expressed as:**

**(Result  $\pm$  Error) unit**



# The uncertainty

Measurement result is expressed as:

**(Result  $\pm$  Error) unit**

□ Estimated uncertainty is written with a  $\pm$  sign; for example:

□  $8.8 \pm 0.1$  cm

□ Percent uncertainty is the **ratio of the uncertainty to the measured value, multiplied by 100:**

□ **Percent uncertainty** =  $\frac{\text{uncertainty value}}{\text{measured value}} \times 100 \%$

The percent uncertainty =  $\frac{\text{Error}}{\text{Result}} \times 100 \%$

**Example:**

The percent uncertainty in the measurement  
 $L = 20.2 \pm 0.4$  cm is:

A	0.5%
B	1%
C	2%✓
D	4%

$$\text{The Percent uncertainty (P.U.)} = \frac{\text{Error}}{\text{Result}} \times 100 \%$$

$$\text{Error} = 0.4 \text{ cm} \quad ; \quad \text{result} = 20.2 \text{ cm}$$

$$\therefore \text{P.U.} = \frac{0.4 \text{ cm}}{20.2 \text{ cm}} \times 100 \% = 1.9801\% \approx 2\%$$

- The uncertainty is generally assumed to be one or a few units in the last digit specified.

**Example:** The percent uncertainty in a measurement  $A = 2.03 \text{ m}^2$  is:

A	0.5%✓
B	1%
C	2%
D	4%

Result =  $2.03 \text{ m}^2$  (with *two decimal places*)

⇒ In this case the **error** is taken as the smallest number with *two decimal places*.

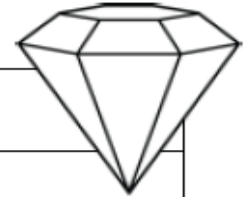
⇒ Error =  $0.01 \text{ m}^2$

$$\text{The Percent uncertainty (P.U.)} = \frac{\text{Error}}{\text{Result}} \times 100 \%$$

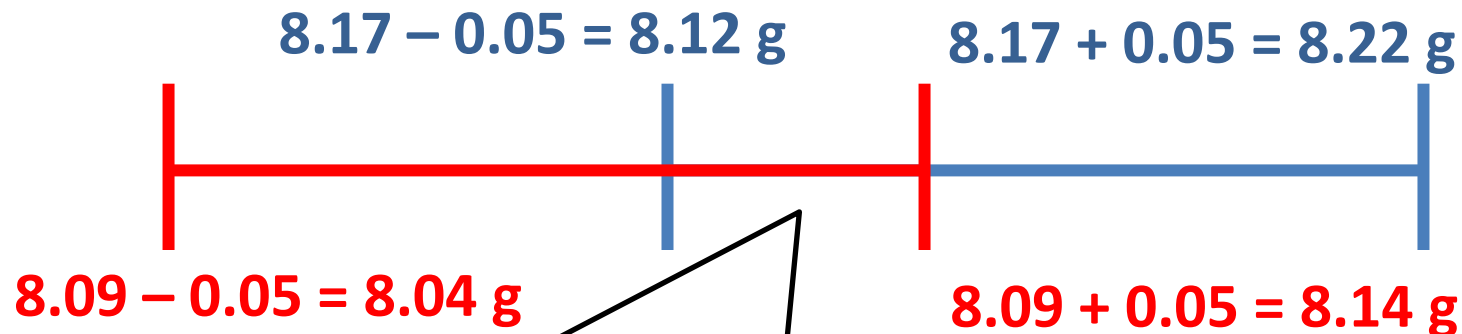
$$\therefore \text{P.U.} = \frac{0.01 \text{ m}^2}{2.03 \text{ m}^2} \times 100 \% \approx 0.5 \%$$

## Conceptual Example: Is the diamond yours?

A scale (ميزان) has  $\pm 0.05$  g accuracy. Weighing a diamond (ماسة) on it gives 8.17 g one day and 8.09 g another day. These two measurements:



A	are unacceptable within the scale's accuracy
B	are acceptable within the scale's accuracy ✓
C	prove that the scale's accuracy is incorrect
D	prove that these are two different diamonds



Results from day one (blue) and day two (red) overlap  
⇒ The two measurements agree

# Significant figure

The number of reliably known digits in a number

For example: 2 cm, 2.0 cm, & 2.00 cm are mathematically the same but experimentally different: *They have different significant figures.*

## Counting rules

**Rule 1:** All nonzero digits (1, 2, 3, ..., 7, 8, 9) are significant figures

Q. determine the number of significant figures in:

**11** and **235.68**

Ans.

$\begin{array}{c} 1 \quad 1 \\ \uparrow \quad \uparrow \end{array}$  Has *two* significant figures

$\begin{array}{c} 2 \quad 3 \quad 5 \quad . \quad 6 \quad 8 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \end{array}$  Has *five* significant figures

## Rule 2:

## Zeros

Trailing zeros (*to the right of a number*)

Zeros in-between digits

Leading zeros (*to the left of a number*)

⇒ **Count**

⇒ **Don't count**

Number *with* decimal point

Number *without* decimal point

⇒ **Count**

⇒ **Don't count**

10.00  
↑ ↑ ↑ ↑

10  
↑ ×

630.  
↑ ↑ ↑

630  
↑ ↑ ×

17.300  
↑ ↑ ↑ ↑ ↑

540000  
↑ ↑ × × × ×

2033  
↑ ↑ ↑ ↑

1001  
↑ ↑ ↑ ↑

0106.22  
× ↑ ↑ ↑ ↑ ↑

0091  
× × ↑ ↑

0.1  
× ↑

0.0051  
× × × ↑ ↑

## Trailing zeros: special case

- A whole number (*without decimal point*) preceded by the word **about** (**approximately, etc ...**)  $\Rightarrow$  **don't count zeros**

About 80 cm  
           $\uparrow$   $\times$

Implying  $\pm 10$  cm uncertainty

About 1300 km  
           $\uparrow$   $\uparrow$   $\times$   $\times$

Implying  $\pm 100$  km uncertainty

- A whole number (*without decimal point*) preceded by the word **precisely** (**accurately, etc ...**)  $\Rightarrow$  **Count zeros**

precisely 80 cm  
           $\uparrow$   $\uparrow$

Implying  $\pm 1$  cm uncertainty

precisely 1300 km  
           $\uparrow$   $\uparrow$   $\uparrow$   $\uparrow$

Implying  $\pm 1$  km uncertainty

Determine (the number of significant digits) of each measurement.

Measurement	(significant digits)
(a) 2642 ft	4
(b) 2005 m	4 (Both zeros are significant.)
(c) 2050 m	3 (Only the first zero is significant.)
(d) 2500 m	2 (No zero is significant.)
(f) 2500.m	4 (Both zeros are significant.)
(g) 34,000 mi	2 (No zeros are significant.)
(h) 15,670,000 lb	4 (No zeros are significant.)
(i) 203.05 km	5 (Both zeros are significant.)
(j) 0.000345 kg	3 (No zeros are significant.)
(k) 75 N	2
(l) 2.3 s	2
(m) 0.02700 g	4 (Only the right two zeros are significant.)
(n) 2.40 cm	3 (The zero is significant.)
(o) 4.050 $\mu\text{s}$	4 (All zeros are significant.)
(p) 100.050 km	6 (All zeros are significant.)
(q) 0.004 s	1 (No zeros are significant.)
(r) $2.03 \times 10^4 \text{ m}^2$	3 (The zero is significant.)
(s) $1.0 \times 10^{-3} \text{ N}$	2 (The zero is significant.)
(t) $5 \times 10^6 \text{ kg}$	1
(u) $3.060 \times 10^8 \text{ m}^3$	4 (Both zeros are significant.)



## Check your understanding

17. The number of significant figures in (23.20) is:

A	1
B	2
C	3
D	4✓

18. The number of significant figures in (0.062) is:

A	1
B	2✓
C	3
D	4

19. The number of decimal places in (0.062) is:

A	1
B	2
C	3✓
D	4

# Significant figure: *mathematical operations*

Carry out intermediate calculations without rounding. Only round the final answer (outcome) according to the following rules:

## 1. Multiplications and divisions

Final result presented with ***significant figures*** similar to that for the number with ***least significant figure*** used in the operation.

$$11.3 \text{ cm} \times \underset{\uparrow}{2}.\underset{\uparrow}{0} \text{ cm} = 22.6 \text{ cm}^2 \xrightarrow{\text{Round}} \underset{\uparrow}{2}\underset{\uparrow}{3} \text{ cm}^2$$

## 2. Addition and subtraction

Final result presented with ***decimal places*** similar to that for the number with ***least decimal places*** used in the operation.

The final result is no more accurate than the least accurate number used.

$$9.300 \text{ cm} + \underset{\uparrow}{0}.\underset{\uparrow}{0}1 \text{ cm} = 9.310 \text{ cm} \xrightarrow{\text{Round}} \underset{\uparrow}{9}.\underset{\uparrow}{3}1 \text{ cm}$$

# 1-4 Measurement and Uncertainty; Significant Figures

- Calculators will not give you the right number of significant figures; they usually give too many but sometimes give too few (especially if there are trailing zeroes after a decimal point).
- The top calculator shows the result of  $2.0 / 3.0$ .
- The bottom calculator shows the result of  $2.5 \times 3.2$ .

## Examples:

$$3.6 - 0.57 = 3.03 = 3.0 \text{ ( Rounding off to one place )}$$

$$4.83 + 2.1 = 6.93 = 6.9$$

$$6.53 + 2 = 8.53 = 9$$

$$8.42 \times 3.0 = 25.26 = 25$$

$$6.00 / 2.0 = 3.0$$



(a)



(b)

## Remember

The approach to **rounding off** is summarized as follows

If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. •

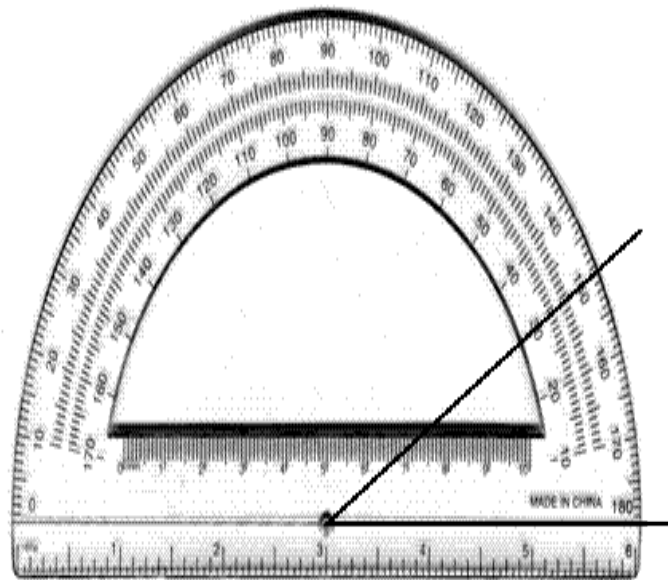
Thus, 1.684 becomes 1.68. (*underestimating*)

If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25. (*overestimating*) •

# Exercises

- The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (a) 14.625 cm<sup>2</sup>; (b) 14.63 cm<sup>2</sup>; (c) 14.6 cm<sup>2</sup>; (d) 15 cm<sup>2</sup>.
- Do 0.00324 and 0.00056 have the same number of significant figures.
- For each of the following numbers, state the number of significant figures and number of decimal places:  
(a) 1.23; (b) 0.123; (c) 0.0123.

# Conceptual example



Using a protractor you measure an angle to be  $30^\circ$ .

- How many significant figures should you quote in this measurements?
- Use a calculator to find the cosine of the angle you measured.

Check your understanding

20. The area of a (10.0 cm  $\times$  6.5 cm) rectangle is correctly given as:

A	65 cm <sup>2</sup> ✓
B	65.0 cm <sup>2</sup>
C	65.00 cm <sup>2</sup>
D	65.000 cm <sup>2</sup>

23. Taking accuracy into account, the difference  $D = A - B$  between two numbers,  $A = 3.6$  and  $B = 0.57$ , is correctly written as:

A	3.03
B	3.00
C	3.003
D	3.0✓

# Scientific Notation(**Powers of 10**)

- Common to express very large or very small numbers using powers of 10 notation.

- A number **greater than 10**  $\implies$  **positive** power of 10

$$39,600 = 3.96 \times 10^4$$

(moved decimal 4 places to left)

- A number **between 0 and 1**  $\implies$  **negative** power of 10

$$0.0021 = 2.1 \times 10^{-3}$$

(moved decimal 3 places to right)

- A number **between 1 and 10**  $\implies$  **zero** power of 10

$$7.33 = 7.33 \times 10^0$$

- Useful for controlling significant figures:

$$39600 \equiv \underset{\uparrow}{3}.\underset{\uparrow}{9}\underset{\uparrow}{6}\underset{\uparrow}{0} \times 10^4 = \underset{\uparrow}{3}.\underset{\uparrow}{9}\underset{\uparrow}{6}\underset{\uparrow}{0}\underset{\uparrow}{0}\underset{\uparrow}{0}\underset{\uparrow}{0} \times 10^4 \approx \underset{\uparrow}{4} \times 10^4$$



# Scientific Notation

- Write the following numbers in full with the correct number of zeros :

- $8.69 \times 10^4 \rightarrow 86900$

- $3.62 \times 10^{-5} \rightarrow 0.0000362$

The **positive (+)** power of ten require moving the decimal point to the **right** .

The **negative (-)** power of ten require moving the decimal point to the **left** .

**EXAMPLE** Write  $7.62 \times 10^2$  in **decimal form**.

$$7.62 \times 10^2 = 762 \quad (\text{Move the decimal point two places to the right}).$$

**EXAMPLE** Write  $6.15 \times 10^{-4}$  in **decimal form**.

$$6.15 \times 10^{-4} = 0.000615$$

(Move the decimal point four places to the left and insert three zeros).

# PROBLEMS

**Write each number in scientific notation.**

- |            |            |          |
|------------|------------|----------|
| 1) 326     | 2) 798     | 3) 2650  |
| 4) 14,500  | 5) 826.4   | 6) 24.97 |
| 7) 0.00413 | 8) 0.00053 | 9) 6.43  |

**Write each number in decimal form.**

- |                       |                          |                          |
|-----------------------|--------------------------|--------------------------|
| 1) $8.62 \times 10^4$ | 2) $8.67 \times 10^2$    | 3) $6.31 \times 10^{-4}$ |
| 4) $5.41 \times 10^3$ | 5) $7.68 \times 10^{-1}$ | 6) $9.94 \times 10^1$    |
| 7) $7.77 \times 10^8$ | 8) $4.19 \times 10^{-6}$ | 9) $4.05 \times 10^0$    |

**The number  $0.00123 \times 10^{-3}$  can be expressed in scientific notation as.....**

# Units, Standards, SI System

- The measurement of any quantity is made relative to a particular **standard** or **unit** .
- The **unit** must be specified along with numerical value of the quantity .
- All measured physical quantities have units.
- Units are **VITAL** in physics!!
- The **SI system of units**:  
**SI** = “**Système International**” (French)  
More commonly called the “**MKS system**” (meter-kilogram-second) or more simply, “**the metric system**”
- For any unit we use, we need to define a **standard**. It is important that standards are readily reproducible

# SI or MKS System

Defined in terms of **standards** (a standard  $\equiv$  *one unit of a physical quantity*) for length, mass, time, ... .

The **Unit** must be specified along with the numerical value of the quantity

- **Length** :

The standard unit of **length** is **Meter (m)** (kilometer = km = 1000 m)

## **Standard meter.**

- The standard meter was originally chosen to be **one ten-millionth of the distance from the Earth's equator to either pole** [a platinum rod to represent length was made].
- Newest definition in terms of speed of light  $\equiv$  **Length of path traveled by light in vacuum during a time interval of (1/299,792,458) of a second .**

- **Time** The standard unit of **time** is the **second (s)**.

### **Standard second.**

- The second was defined as **1/86,400** of a mean solar day.
- Newest definition  $\equiv$  **time required for 9,192,631,770 oscillations of radiation emitted by cesium atoms .**

- **Mass** The standard unit of **mass** is the **Kilogram (kg)**  
(kilogram = kg = 1000 g)

### **Standard kg.**

- A particular platinum-iridium cylinder whose mass is defined as exactly 1 kg
- When dealing with atoms and molecules, we usually use the **unified atomic mass unit (u)**.

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg.}$$

<b>Unit</b>	<b>Original standard</b>	<b>Current standard</b>
meter (length)	$\frac{1}{10\,000\,000}$ distance from equator to North Pole	the distance traveled by light in a vacuum in $3.33564095 \times 10^{-9}$ s
kilogram (mass)		the mass of a specific platinum-iridium alloy cylinder
second (time)	$\left(\frac{1}{60}\right) \left(\frac{1}{60}\right) \left(\frac{1}{24}\right) =$ 0.000 011 574 average solar days	9 192 631 770 times the period of a radio wave emitted from a cesium-133 atom

# Base versus Derived Quantities

- Physical quantities can be divided into:

- **Based quantities**

**It is the seven quantities in the SI system and must be define in terms of standard**

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

# SI Derived Quantities and Units

All physical quantities are *defined* in terms of the *base quantities*

- Both the **quantity** and its **unit** are derived from a combination of base units, using a defining equation.

**Example: Derived units for *speed, acceleration and force*:**

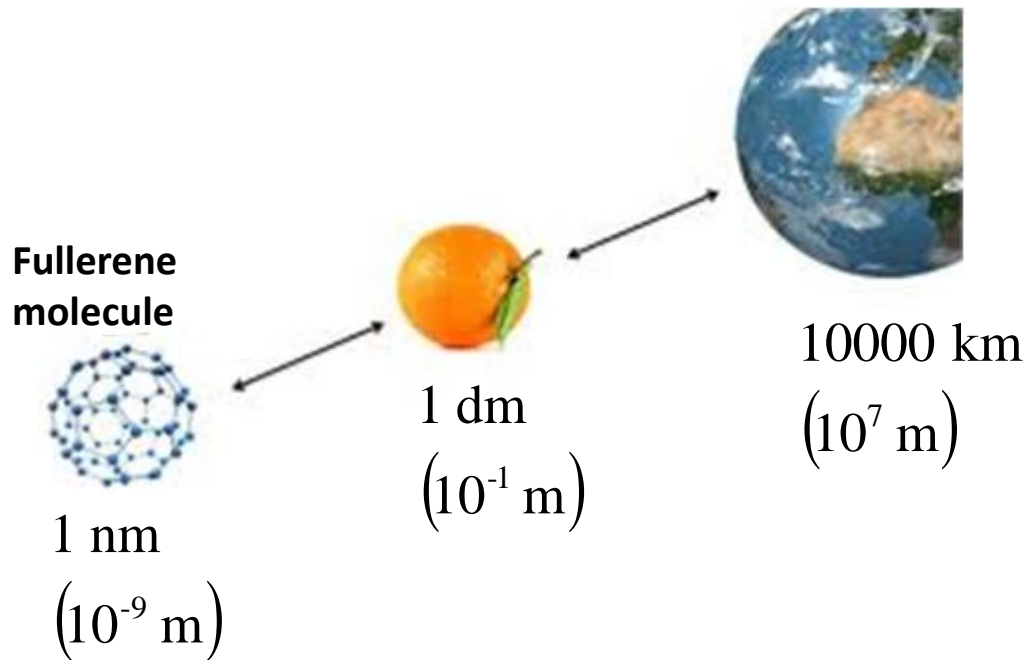
$$\text{Speed (m/s)} = \frac{\text{Distance (m)}}{\text{Time (s)}}$$

$$\text{Acceleration (m/s}^2\text{)} = \frac{\Delta \text{ velocity (m/s)}}{\text{Time (s)}}$$

$$\text{Force (Newton, N)} = \text{Mass (kg)} \times \text{Acceleration (m/s}^2\text{)}$$



# Larger & smaller units defined from SI standards by powers of 10 & Greek prefixes



Prefix	Abbreviation	Value
exa	E	$10^{18}$
peta	P	$10^{15}$
tera	T	$10^{12}$
giga	G	$10^9$
mega	M	$10^6$
kilo	k	$10^3$
hecto	h	$10^2$
deka	da	$10^1$
deci	d	$10^{-1}$
centi	c	$10^{-2}$
milli	m	$10^{-3}$
micro <sup>†</sup>	$\mu$	$10^{-6}$
nano	n	$10^{-9}$
pico	p	$10^{-12}$
femto	f	$10^{-15}$
atto	a	$10^{-18}$

The number  $4.436 \times 10^4$  km can be written as:

- A. 4.436 Mm      B. 4436 km  
C. 4436 Mm      D. 44.36 Mm

# Other Systems of Units

- **CGS** (centimeter-gram-second) system
  - **Centimeter** = 0.01 meter
  - **Gram** = 0.001 kilogram
- **British** (foot-pound-second) system
  - Our “everyday life” system of units
  - Still used in some countries like USA
  - British engineering system **has force instead of mass as one of its basic quantities**

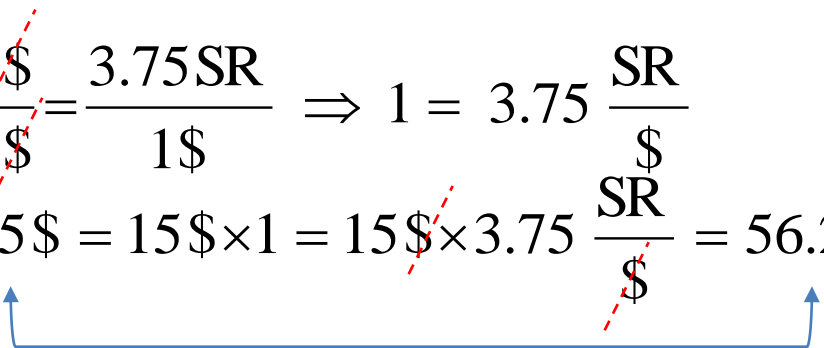
# Converting Units

Unit conversions always involve a **conversion factor**.

Suppose you are to convert 15 US Dollar (\$) into Saudi Riyal (SR) :

1<sup>st</sup> Find conversion factor: 1 \$ = 3.75 SR

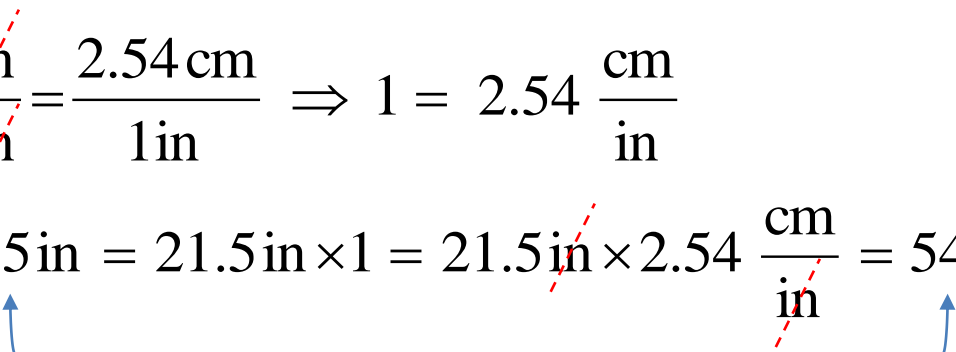
$$2^{\text{nd}} \quad \frac{1\cancel{\$}}{1\cancel{\$}} = \frac{3.75\text{SR}}{1\$} \Rightarrow 1 = 3.75 \frac{\text{SR}}{\$}$$

$$3^{\text{rd}} \quad 15\$ = 15\$ \times 1 = 15\cancel{\$} \times 3.75 \frac{\text{SR}}{\cancel{\$}} = 56.25\text{SR}$$


Likewise convert **21.5 inches (in)** into **cm** :

1<sup>st</sup> Conversion factor: 1 in = 2.54 cm

$$2^{\text{nd}} \quad \frac{1\cancel{\text{in}}}{1\cancel{\text{in}}} = \frac{2.54\text{cm}}{1\text{in}} \Rightarrow 1 = 2.54 \frac{\text{cm}}{\text{in}}$$

$$3^{\text{rd}} \quad 21.5\text{in} = 21.5\text{in} \times 1 = 21.5\cancel{\text{in}} \times 2.54 \frac{\text{cm}}{\cancel{\text{in}}} = 54.6\text{cm}$$


49. A distance of 10 ft. is equal to:

A	305 m
B	305 cm ✓
C	30.5 cm
D	30.5 m

*Hint:*

**1 ft = 12 in**

**and**

**1 in = 2.54 cm**

52. The maximum capacity in liters of a 3-m<sup>3</sup> water tank  
(خزان) is:

A	30 L
B	3000 L ✓
C	300 L
D	3 L

*Hint:*

**1 m<sup>3</sup> = 1000 L**

## Working Examples

1. What is the elevation, in feet, of an elevation of 8000 m?(1ft=12in.)

2. A silicon chip has an area of 1.25 square inches. Express this in square centimeters.

3. Posted speed limit is 55 miles per hour, what is this speed in kilometer per hour? (1 mile = 5280 ft)

4. Would a driver travelling at 15 m/s in a 35 mi/h zone be exceeding the speed limit?

## EXAMPLE 1.4

Area of a semiconductor chip. A silicon chip has an area of 1.25 square inches. Express this in square centimeters.

$$1.25 \text{ in.}^2 = (1.25 \text{ in.}^2) \left( 2.54 \frac{\text{cm}}{\text{in.}} \right)^2 = (1.25 \cancel{\text{in.}^2}) \left( 6.45 \frac{\text{cm}^2}{\cancel{\text{in.}^2}} \right) = 8.06 \text{ cm}^2.$$

## EXAMPLE 1.5

Speeds. Where the posted speed limit is 55 miles per hour (mi/h or mph), what is this speed (a) in meters per second (m/s) and (b) in kilometers per hour (km/h)?

$$1 \text{ mi} = (5280 \cancel{\text{ft}}) \left( 12 \frac{\cancel{\text{in.}}}{\cancel{\text{ft}}} \right) \left( 2.54 \frac{\cancel{\text{cm}}}{\cancel{\text{in.}}} \right) \left( \frac{1 \text{ m}}{100 \cancel{\text{cm}}} \right) = 1609 \text{ m.}$$

$$55 \frac{\text{mi}}{\text{h}} = \left( 55 \frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \right) \left( 1609 \frac{\text{m}}{\cancel{\text{mi}}} \right) \left( \frac{1 \cancel{\text{h}}}{3600 \text{ s}} \right) = 25 \frac{\text{m}}{\text{s}},$$

$$55 \frac{\text{mi}}{\text{h}} = \left( 55 \frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \right) \left( 1.609 \frac{\text{km}}{\cancel{\text{mi}}} \right) = 88 \frac{\text{km}}{\text{h}}.$$

# Order of Magnitude; Rapid Estimating

- Sometimes, we are interested in only an approximate value for a quantity. We are interested in obtaining rough or **order of magnitude estimates**.
- **Order of magnitude estimates:** Made by rounding off all numbers in a calculation to 1 significant figure, along with power of 10.
  - Can be accurate to within a factor of 10 (often better)

The order of magnitude of

$$(a) \quad 2800 = \quad \quad \quad : \quad 2.8 \times 10^3 \approx 1 \times 10^3 = \boxed{10^3}$$

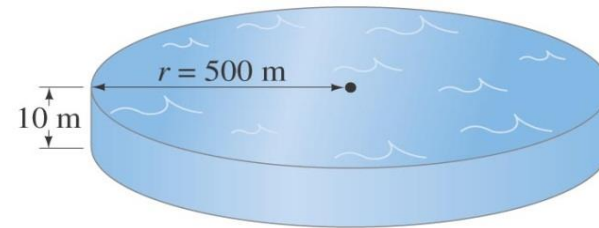
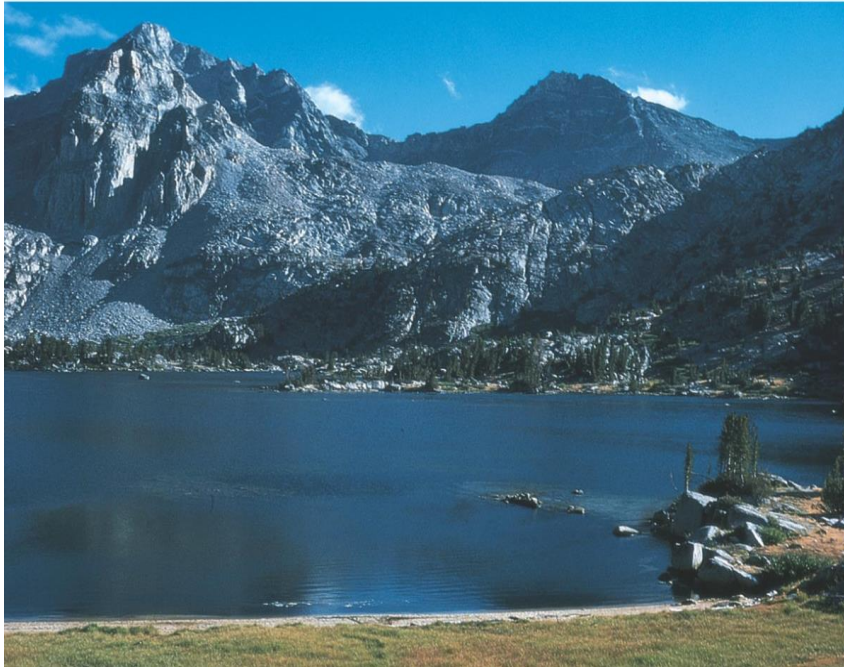
$$(b) \quad 86.30 \times 10^2 = \quad 8.630 \times 10^3 \approx 10 \times 10^3 = \boxed{10^4}$$

$$(c) \quad 0.0076 = \quad \quad \quad 7.6 \times 10^{-3} \approx 10 \times 10^{-3} = \boxed{10^{-2}}$$

$$(d) \quad 15.0 \times 10^8 = \quad \quad \quad 1.5 \times 10^9 \approx 1 \times 10^9 = \boxed{10^9}$$

# 1-7 Order of Magnitude: Rapid Estimating

## Example 1-5: Volume of a lake.



**Estimate** how much water there is in a particular lake, which is roughly circular, about **1 km across**, and you guess it has an average **depth of about 10 m**.



**Volume of a lake.** Estimate how much water there is in a particular lake, Figure 1.8a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

The volume  $V$  of a cylinder is the product of its height  $h$  times the area of its base:  
 $V = h\pi r^2$ , where  $r$  is the radius of the circular base. The radius  $r$  is  $\frac{1}{2}$  km = 500 m, so the volume is approximately

$$V = h\pi r^2 \approx (10 \text{ m}) \times (3) \times (5 \times 10^2 \text{ m})^2 \approx 8 \times 10^6 \text{ m}^3 \approx 10^7 \text{ m}^3,$$

where  $\pi$  was rounded off to 3. So the volume is on the order of  $10^7 \text{ m}^3$ , ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate ( $10^7 \text{ m}^3$ ) is probably better to quote than the  $8 \times 10^6 \text{ m}^3$

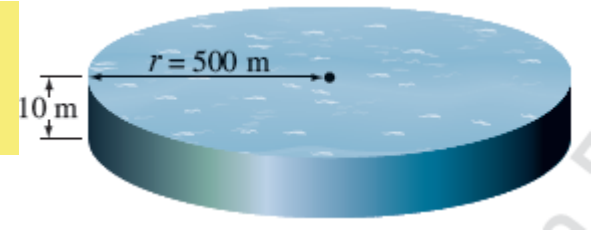


**Total number of heartbeats.** Estimate the total number of beats a typical human heart makes in a lifetime.

If an average person lives 70 years  $\approx 2 \times 10^9$  s,

$$\left(80 \frac{\text{beats}}{\text{min}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) (2 \times 10^9 \text{ s}) \approx 3 \times 10^9,$$

or 3 trillion.



56. In the world, the 14 highest peaks are between 8000 m and 9000 m high. The order-of-magnitude of their height (ارتفاع) is:

A	$1 \times 10^4 \text{ m}$ ✓
B	$0.1 \times 10^4 \text{ m}$
C	$2 \times 10^4 \text{ m}$
D	$10 \times 10^4 \text{ m}$

58. The thickness (سمائة) of a 200-page book is 1.0 cm. The thickness of one sheet of this book can be estimated as:

A	0.001 mm
B	0.01 mm
C	0.1 mm✓
D	1 mm

You measure that a book with 300 sheets of paper is 3-cm thick. This means that the thickness of one sheet of this book is..... $\mu\text{m}$

**Explanation:**

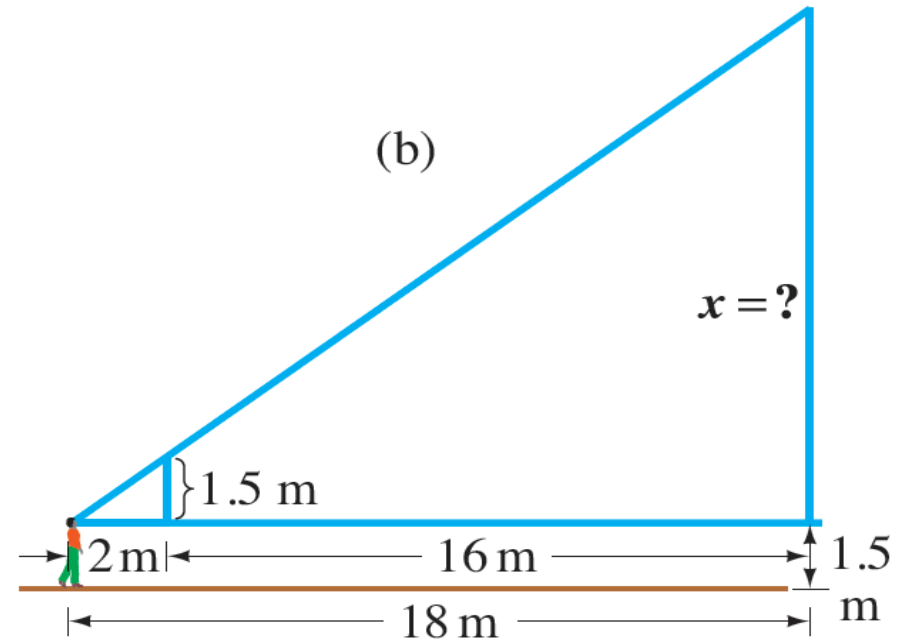
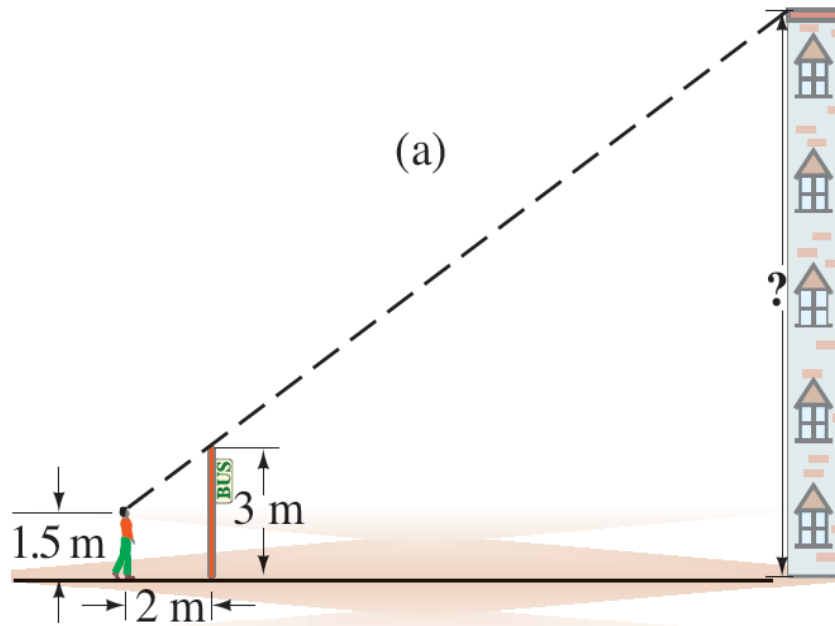
9000 m ~ 10000 m =  $10^4 \text{ m}$   
 $\vdots$   
 8500 m ~ 9000 m ~ 10000 =  $10^4 \text{ m}$   
 $\vdots$   
 8000 ~ 10000 =  $10^4 \text{ m}$



59. If an average human lives for 70 years, and if the heartbeat rate is 80 beats/min, the number of heartbeats in a lifetime can be estimated as:

A	$3 \times 10^6$
B	$3 \times 10^7$
C	$3 \times 10^8$
D	$3 \times 10^9$ ✓

**Height by triangulation.** Estimate the height of the building shown in Figure 1.10, by “triangulation,” with the help of a bus-stop pole and a friend.



# Dimensions and Dimensional Analysis

- The dimension of a physical quantity is the type of units or **base quantities** that make it up.

<u>Base quantity</u>	<u>Dimension abbreviation</u>
Length	[L]
Time	[T]
Mass	[M]
...	...

$$\text{Dimension of the velocity \& speed} = [V] = \frac{[L]}{[T]}$$

$$\text{Dimension of the acceleration} = \frac{[L]}{[T^2]}$$

- Dimensional analysis is the checking of dimensions of all quantities in an equation to ensure that those which are added, subtracted, or equated have the same dimensions.**

# Dimensional analysis:

**Example:**

$$\underbrace{V}_{\text{Final velocity}} = \underbrace{V_o + \frac{1}{2} a \cdot t^2}_{\text{Initial velocity}} \quad \text{Left Hand Side (LHS)} \quad \text{Right Hand Side (RHS)}$$

$$\underbrace{\frac{[L]}{[T]}}_{\text{LHS dimension}} \stackrel{?}{=} \underbrace{\frac{[L]}{[T]} + \frac{[L]}{[T^2]} \cdot [T^2]}_{\text{RHS dimension}} = \underbrace{\frac{[L]}{[T]} + [L]}_{\text{RHS dimension}}$$

⇒ LHS dimension ≠ RHS dimension

⇒ **The equation is *incorrect***

If LHS dimension = RHS dimension

⇒ **The equation is *dimensionally correct* (But could be physically incorrect)**

Ex. Which of the following is dimensionally correct

$$T = 2\pi\sqrt{l/g} \text{ or } T = 2\pi\sqrt{g/l}$$

Where  $g$  is the acceleration due to gravity

$$T = 2\pi\sqrt{l/g}$$

$$[T] = \sqrt{\frac{[L]}{[L/T^2]}} = \sqrt{[T^2]} = [T]$$

$$T = 2\pi\sqrt{g/l}$$

$$[T] \neq \sqrt{\frac{[L/T^2]}{[L]}} = \sqrt{\frac{1}{[T^2]}} = \frac{1}{[T]}$$

60. The dimensions of volume are:

A	$L^3$
B	$L^2$
C	$L^3/T^2$
D	$L^2 T^{-1}$

61. The dimensions of force are:

A	$L M T$
B	$L M T^{-2}$
C	$L^3 M^2/T^2$
D	$L^2 M T^{-1}$

62. \* Which of the following is dimensionally correct?

A	speed = acceleration / time
B	distance = speed / time
C	force = mass $\times$ acceleration
D	density = mass $\times$ volume

The equation: speed = (acceleration  $\times$  time<sup>n</sup>) is dimensionally correct if n equals.....