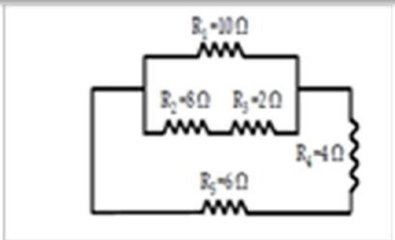
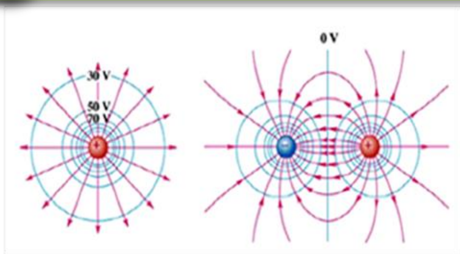
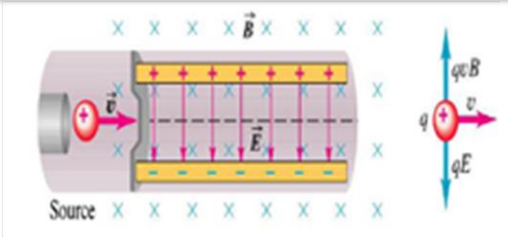
Physics-2







Ch-4

The electric potential

يوسف زويل

Best wishes and good Luck

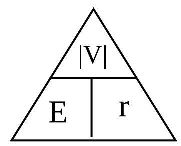
00966502047005

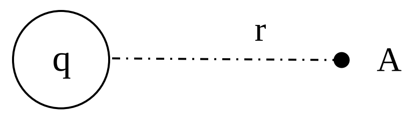
The electric potential

الجهد الكهربي V

في مجال شحنة واحدة

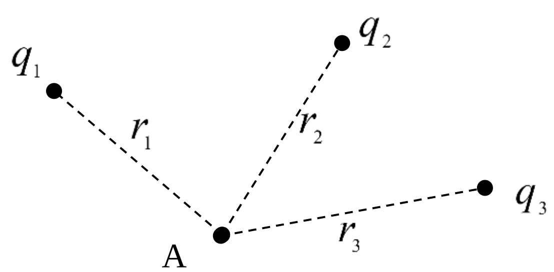
$$V_A = K\frac{q}{r}$$ فولت V





في مجال عدة شحنات

$$V_A = K\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_2}+.....\right]$$ فولت V



ملحوظة: هامة

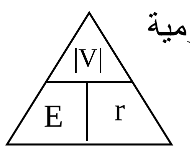
(1) في حالة حساب الجهد يتم التعويض عن كل شحنة بإشارتها.

(2) الجهد كمية قياسية لا يوجد له اتجاه لذا لا توجد زوايا ولا تحليل.

(3) وحدات قياس الجهد هي الفولت V ومضاعفاته (KV, MV, GV).

(4) جهد الشحنة الموجبة (+) وجهد الشحنة السالبة (-).

(5) يمكن حساب شدة المجال عند نقطة في مجال شحنة بمعلومية الجهد من العلاقة



(6) إذا علم إحداثيات كل من الشحنة والنقطة فإن البعد بينهما يكون

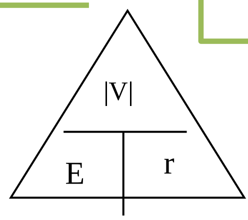
$$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

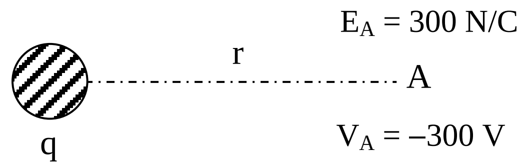
Ex:1 At a certain distance from a point charge the magnitude of the electric field is 300 N/C and the potential is -300 V. The distance of the point from the charge is:

a) 8 m b) 3 m c) 10 m d) 5 m

Solution

r ??



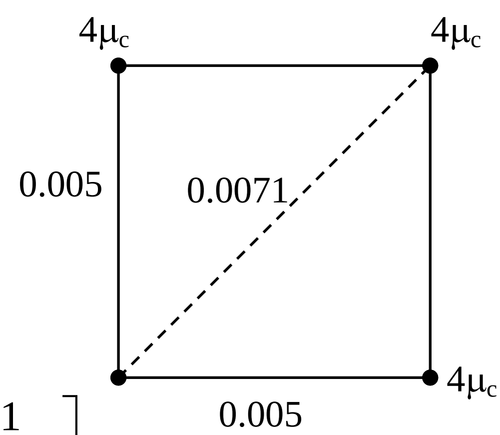


$$r = \frac{|V|}{E} = \frac{3000}{3000} = 10 \text{ m}$$

Ex:2 Three point charges each of charge is $4\mu c$ are located at the corners of square of side length 0.5 cm. The electric potential at the forth corner of the square is:

a) 3×10^{8} V b) 1.95×10^{7} V c) 1.59×10^{7} V d) 5.19×10^{7} V

Solution



$$V_A = K\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

$$= K\,q\left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right]$$

$$= 9\times10^{9}\times4\times10^{-6}\left[\frac{1}{0.005} + \frac{1}{0.0071} + \frac{1}{0.005}\right]$$

$$= 1.95\times10^{7} \text{ V}$$

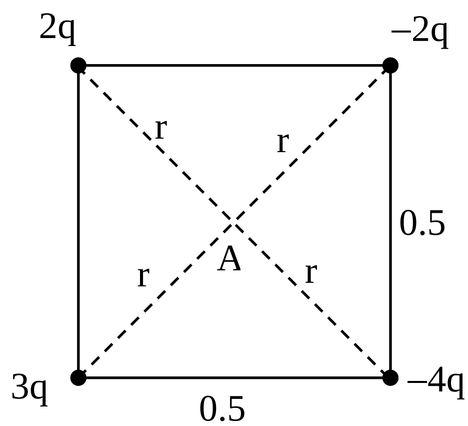
Ex:3 Four point charges are located at the corners of a square of side 0.5 m in length. The electric potential at the center of this square is:

a) Zero
b) -2.6×10^{4} V
c) 2.6×10^{4} V
d) 1.13×10^{5} V

Solution

* المطلوب حساب V عند مركز المربع فتكون كل الأبعاد متساوية.

$r_1 = r_2 = r_3 = r_4 = r$ (تحسب من فيثاغورث)



القطر $2r = \sqrt{0.5^2 + 0.5^2} = 0.707$

$$r = \frac{0.707}{2} = 0.35 \text{ m}$$

$$V_A = \frac{K}{r}[q_1 + q_2 + q_3 + q_4]$$

$$= \frac{K}{r}[2q - 2q - 4q + 3q]$$ (لاحظ أن الشحنة بإشارتها والجمع جبري)

$$= \frac{-Kq}{r} = \frac{-9\times10^{9}\times1\times10^{-6}}{0.35}$$

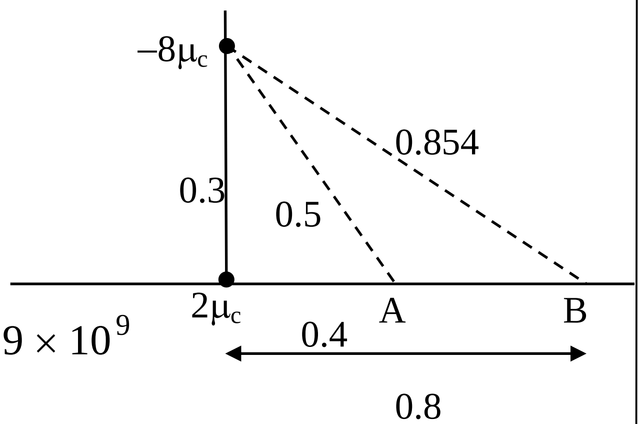
$$V = -2.6\times10^{4} \text{ V}$$

Ex:4 A $2\mu c$ point charge is located at the origin, and a second charge of $-8\mu c$ is placed on the y-axis at y = 30 cm. Point A and B are on the x-axis at 40 cm, 80 cm respectively the potential difference $(V_A - V_B)$ is:

a) $3\times10^4 V$ b) $-3\times10^4 V$

c) $4\times10^4 V$ d) $-4\times10^4 V$

Solution



$$V_A = K\left[\frac{q_1}{r_{1_A}} + \frac{q_2}{r_{2_A}}\right]$$

$$= K\left[\frac{2\times10^{-6}}{0.4} + \frac{(-8\times10^{-6})}{0.5}\right] \times 9\times10^9$$

$$= -9.9\times10^4 \quad V$$

$$V_B = K\left[\frac{q_1}{r_{1_A}} + \frac{q_2}{r_{2_A}}\right]$$

$$=9\times10^9\left[\frac{2\times10^{-6}}{0.8} + \frac{8\times10^{-6}}{0.85}\right]$$

$$= -6.2\times10^4 \quad V$$

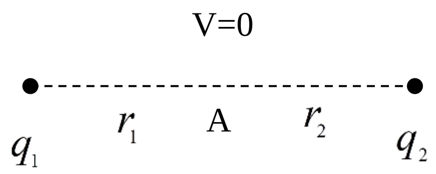
$$V_A - V_B$$

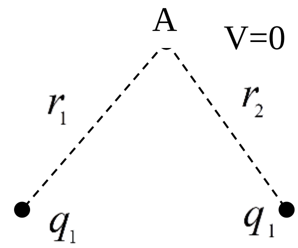
$$= (-9.9\times10^4) - (-6.2\times10^4)$$

$$= -3.7\times10^4 \quad V$$

$$= -4\times10^4 \quad V$$

*** انعدام الجهد الكهربي V=0 عند نقطة في مجال شحنتين:-**





$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

ملحوظة:

(1) لكي ينعدم الجهد عند نقطة في مجال شحنتين لابد أن تكون الشحنات مختلفة (- ، +).

(2) لا يشترط أن تكون نقطة انعدام الجهد على استقامة الشحنتين. أو خارجهما أو داخلهما لكن الأهم أن يكون $V_1 = V_2$ **عند تلك النقطة.**

(3) إذا كانت نقطة انعدام الجهد على استقامة الشحنتين فلابد أن تكون بينهما وأقرب إلى الأقل قيمة $\frac{q_1}{x} = \frac{q_2}{r - x}$.

Ex:5 A $20\mu c$ point charge is at the origin and $-15\mu c$ is at (2,0) m. The point which on x-axis where the total electric potential is Zero is:

a) $(-2,0)$ b) $(1.14,0)$ c) $(-1.14,0)$ d) $(1,0)$

Solution

$q_1 = 20\times10^{-6}\ C$ (0, 0)

$q_2 = -15\times10^{-6}\ C$ (2, 0)

$r = 2\ m$

النقطة المطلوبة على استقامة الشحنتين

∴ لابد أن تكون بينهما.

$$\frac{|q_1|}{x} = \frac{|q_2|}{r-x}$$

$$\frac{20\times10^{-6}}{x} = \frac{15\times10^{-6}}{2-x}$$

$$4(2-x) = 3x$$

$$8-4x = 3x$$

$$8 = 7x \quad \therefore\ x = \frac{8}{7} = 1.14\ m$$

∴ النقطة هي (1.14, 0)

حساب شدة المجال من دالة الجهد الكهربي مع الازاحة

بالاشتقاق

دالة جهد في x فقط	$V(x) \Rightarrow \quad E = -\frac{\partial v}{\partial x}\,i$
دالة جهد في y فقط	$V(y) \Rightarrow \quad E = -\frac{\partial v}{\partial y}\,j$
دالة جهد في z فقط	$V(x) \Rightarrow \quad E = -\frac{\partial v}{\partial z}\,k$
دالة جهد في x, y, z	$V(x,y,z) \Rightarrow \quad E = -\frac{\partial v}{\partial x}\,i - \frac{\partial v}{\partial y}\,j - \frac{\partial v}{\partial z}\,k$

Ex:6 If the electric potential V is a function of x-coordinate only and is given by $V(x) = (a - bx)i$ where a = 100 V and b = 25V/m. The electric field is:

a) $(25\ V/m)\ i$ b) $(23\ V/m)\ i$

c) $(25\ V/m)\ i$ d) $(18\ V/m)\ j$

Solution

$$V = (a - b\,x)\ i$$

$$E = -\frac{\partial v}{\partial x}\, i = -(0 - b)\, i = -(0 - 25) = (25\ \ N/m)\ i$$

Ex:7 The electric dipole lies a long the Z-axis with its center at the origin. The electric potential V due to the dipole at a point A on the Z-axis is given by $V(Z) = KPZ^{-2}$. The electric field at A is:

a) $2KPZ\ \ j$ b) $-2KPZ^{-2}\ \ k$ c) $2KPZ^{-3}\ \ k$ d) $2KPZ^{-3}\ \ j$

Solution

$$V(z) = KPZ^{-2}$$

$$E = -\frac{\partial v}{\partial z}\, k = -(-2\,K\,P\,Z^{-3})$$

$$= (2\,K\,p\,Z^{-3}\ \ V/m)\ k$$

Ex:8 The electric potential in a certain region is given by $V = ax^2 + bx + c$ where $a = 12 V/m^2$, $b = -10 V/m$, $c = 26 V$. The magnitude of the electric field at x = 2 m is:

a) $40 V/m$ b) $38 V/m$

c) $15 V/m$ d) $23 V/m$

Solution

دالة الجهد $V = a x^2 + b x + c$

$E = -(2 a x + b)$ $(x = 2,\ a = 12,\ b = -10)$

$= -(2 \times 12 \times 2 + (-10))$

$= -(48 - 10)$

$= -38$ V/m $E = 38$ V/m

الجهد الناتج عن الحلقة المشحون

Potential due to a charged ring

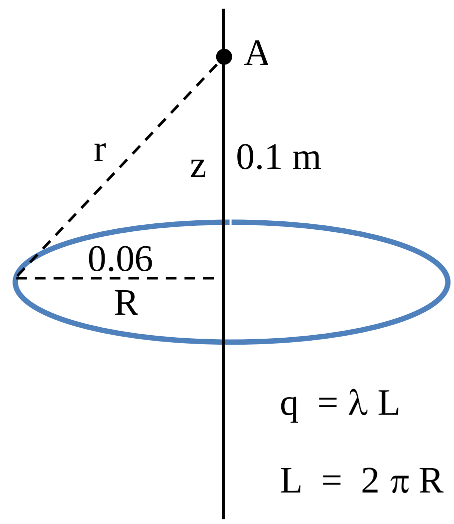
Ex:9 A ring of radius 6 cm and has a charge density of $4\mu c/m$. The electric potential at a point on the axis of the ring at 10 cm from the center is:

a) 1.43×10^{5} V   b) 2.13×10^{5} V   c) 1.25×10^{5} V   d) 1.13×10^{5} V

Solution

$$r = \sqrt{R^2 + Z^2}$$

$$= \sqrt{0.06^2 + 0.1^2} = 0.12 \text{ m}$$



$$q = \lambda L$$

$$= \lambda\, 2\,\pi\, R$$

$$= 4\times10^{-6}\times2\times3.14\times0.06$$

$$= 1.51\times10^{-6} \text{ C}$$

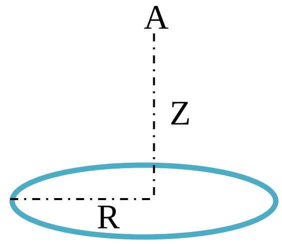
$$V = K\frac{q}{r} = 9\times10^{9}\ \frac{1.51\times10^{-6}}{0.12}$$

$$= 1.13\times10^{5} \text{ V}$$

ملحوظة: عند مركز الحلقة الجهد $V = K\dfrac{q}{r}$ لكن $E=0$

الجهد الناتج عن القرص المشحون

Potential due to a charged disk



$$V_A = \frac{\sigma}{2\in_0}\left[\sqrt{Z^2+R^2} - Z\right]$$

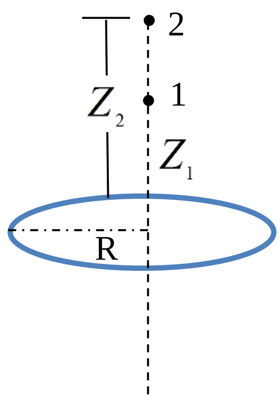
وعند المركز تكون $Z = 0$ ويكون

$$V_c = \frac{\sigma}{2\in_0}.R$$

لا تنسى أن E للقرص هي:

$$E_A = \frac{\sigma}{2\in_0}\left(1 - \frac{Z}{\sqrt{Z^2+R^2}}\right)$$

ملحوظة:-



لنفس القرص

$$\frac{v_1}{v_2} = \frac{\sqrt{Z_1^2+R^2} - Z_1}{\sqrt{Z_2^2+R^2} - Z_2}$$

Ex:10 The electric potential at the center of uniformly charged disc of radius R is 30 V. The potential at a point on the axis of the disc at a distance of $R\sqrt{3}$ is:

a) 10 V b) 12 V c) 8 V d) 13 V

Solution

$Z_1 = 0$ $E_1 = 30\text{ V}$

$Z_2 = R\sqrt{3}$ $E_2 = ??$

$$\frac{V_1}{V_2} = \frac{\sqrt{Z_1^2 + R^2} - Z_1}{\sqrt{Z_2^2 + R^2} - Z_2}$$

$$\frac{30}{V_2} = \frac{\sqrt{0 + R^2} - 0}{\sqrt{(R\sqrt{3})^2 + R^2} - R\sqrt{3}}$$

$$\sqrt{(R\sqrt{3})^2 + R^2}$$

$$= \sqrt{3R^2 + R^2}$$

$$= \sqrt{4R^2} = 2R$$

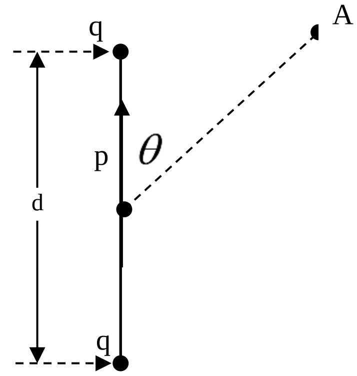
الجهد الناتج عن ثنائي القطب

Potential due to the electric dipole

$$\text{Dipole moment} \rightarrow P = qd$$

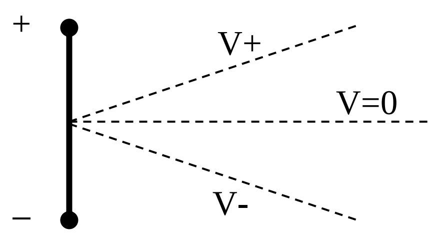
$$V_A = K\frac{P}{r^2}\text{Cos}\theta$$

$$= \frac{Kqd}{r^2}\text{Cos}\theta$$



لاحظ أنه إذا كانت $\theta = 90$

فإن $V = \begin{cases} + & \theta < 90 \\ 0 & \theta = 90 \\ - & \theta > 90 \end{cases}$



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Ex:11 An electric dipole is of an absolute charge q =12nc if the distance between dipole charges is 3cm, then the potential at a point at a distance 15 cm. making an angle 40^o measured from the dipole axis is:

a) -33.5 V b) $+620$ V c) $+110.3$ V d) -77 V

Solution

$q = 12 \times 10^{-9}$ C
$d = 0.03$ m
$r = 0.15$ m
$\theta = 40^o$

$$V = K\frac{Pd}{r^2}\cos\theta$$

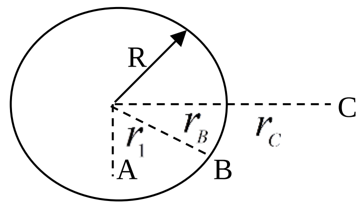
$$= 9\times10^9 \frac{12\times10^{-9}\times0.03}{(0.15)^2}\cos 40$$

$$= 110.3 \quad V$$

الجهد الناتج عن الكرة الموصلة المشحونة

Conducting Sphere

$$V = \begin{cases} K\dfrac{q}{r} & r \le R \\ K\dfrac{q}{r} & r \ge R \end{cases}$$



$$\left[q = \sigma . 4\pi R^2\right]$$

وفي الشكل:

$$\left[V_A = K\frac{q}{R} \;,\; V_B = K\frac{q}{R} \;,\; V_C = K\frac{q}{r}\right]$$

$$r < R \quad | \quad r = R \quad | \quad r > R$$

Ex:12 A conducting sphere of radius 10 cm and charge of $20\mu c$. The electric potential at the center of the sphere is:

a) 3 MV b) 1.8 MV c) 8.1 MV d) 10 MV

Solution

$R = 0.1\ m \qquad r = 0 < R \qquad q = 20 \times 10^{-6}\ C$

$$V = K\frac{q}{R} = 9 \times 10^{9}\ \frac{20 \times 10^{-6}}{0.1} = 1.8 \times 10^{6}\ V$$

$$= 1.8\ MV$$

Ex:13 A conducting sphere of radius 10 cm and has an electric potential at the surface $-3000V (V = 0 \text{ at } \infty)$. The surface charge density at the sphere is:

a) $3.4\times10^{-5} c/m^2$ b) $2.7\times10^{-7} c/m^2$

c) $8.1\times10^{-7} c/m^2$ d) $-2.7\times10^{-7} c/m^2$

Solution

$R = 0.1\ m$ $V = -3000\ V$ $r = R$

$$V = K\frac{q}{R} \Rightarrow q = \frac{VR}{K} = \frac{-3000\times0.1}{9\times10^{9}}$$

الشحنة سالبة لأن V سالبة $= -3.33\times10^{-8}\ C$

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} = \frac{-3.33\times10^{-8}}{4\times3.14\times0.1^2} = -2.7\times10^{-7}\ c/m^2$$

Ex:14 Consider two concentric (then and conducting) spherical shells. The inner has a radius a = 10 cm and a charge of 8 nc. The outer has a radius b=30 cm and a charge of -15 nc. The electric potential on the surface of the inner shell is:

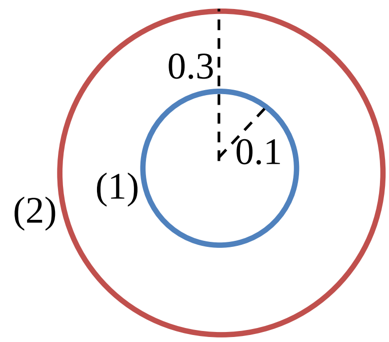
a) 300 V b) 270 V c) 168 V d) 200 V

Solution

$$V = V_1 + V_2$$

$$= K\frac{q_1}{a} + K\frac{q_2}{b}$$

$$(r = a) \qquad (r < b)$$



$$= 9\times10^{9}\left[\frac{8\times10^{-9}}{0.1} + \frac{15\times10^{-9}}{0.3}\right]$$

$$= 270 \quad V$$

ملحوظة: إذا كان

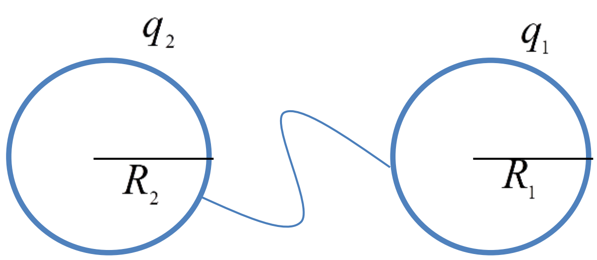
$a < r < b$ فان

$$V = K\left[\frac{q_1}{r} + \frac{q_2}{b}\right]$$

$$r > a \qquad r < b$$

الجهد المشترك لكرتين

Common potential of two spheres



* إذا تم توصيل الكرتين بسلك توصيل رفيع فإن الشحنة تنتقل من الكرة الأعلى جهد إلى الأقل جهد حتى يتساوى الجهد عليهما و عندها

$$\boxed{\frac{q_1'}{R_1} = \frac{q_2'}{R_2}} \quad \rightarrow \quad (1)$$

بحيث أن:

$$\boxed{q_1' + q_2' = q_1 + q_2} \quad \rightarrow \quad (2)$$

ويكون الجهد المشترك بعد التوصيل و الوصول إلى الاتزان:

$$V_{common} = K\frac{q_1'}{R_1} = K\frac{q_2'}{R_2}$$

ملحوظة: بحل المعادلتين 1 ، 2 يمكن إيجاد $q_1' \,,\, q_2'$

Ex:15 Two conducting sphere are very far apart. The smaller sphere carries a total charge of qμc. The larger sphere has a radius twice that of the smaller sphere and is uncharged. After the two spheres are connected by a thin conducting wire, the charge on the smaller and the larger spheres respectively are:

a) 3μc, 6μc b) 6μc, 3μc

c) 2μc, 4μc d) 4μc, 2μc

Solution

$R_1 = R$ $q_1 = 9 \times 10^{-6}$ C $q'_1 = ??$

$R_2 = 2R$ $q_2 = 0$ $q'_2 = ??$

$$q_1 + q_2 = q'_1 + q'_2$$

$$9 \times 10^{-6} = q'_1 + q'_2 \rightarrow (1)$$

$$\frac{q'_1}{R_1} = \frac{q'_2}{R_2}$$

$$\frac{q'_1}{R} = \frac{q'_2}{2R}$$

$$\therefore q'_2 = 2q'_1 \rightarrow (2)$$

بالتعويض من (2) في (1)

$$9 \times 10^{-6} = q'_1 + 2q'_1$$

$$3q'_1 = 9 \times 10^{-6}$$

$$q'_1 = 3 \times 10^{-6} = 3 \ \mu c$$

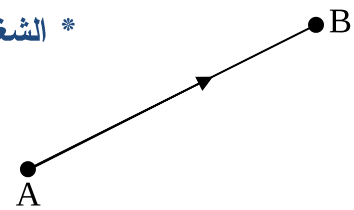
$$q'_2 = 2q'_1$$

$$= 6 \times 10^{-6} = 6 \ \mu c$$

electric potential difference between A , B

* The work required to bring q (1C) from A to B:

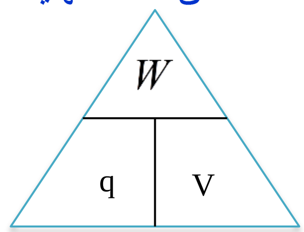
* الشغل اللازم لنقل الشحنة q من النقطة A إلى النقطة B:



$$W = q\,(V_B - V_A)$$

ملحوظة: عند نقل الشحنة من ما لا نهاية $V_\infty = 0$ إلى نقطة جهدها V يكون:

$$\boxed{W = q\,V} \rightarrow$$



Ex:16 An external agent brings a charge of $4\mu c$ from infinity to a point where the electric potential is 24×10^{6} V. The work done by the agent is:

a) 30 J b) 90 J c) 96 J d) 100 J

Solution

$q = 4\times10^{-6}$

$V_2 = 24\times10^{6}$ V

$V_{1_\infty} = 0$

$W = q\,V$

$= 4\times10^{-6}\times24\times10^{6}$

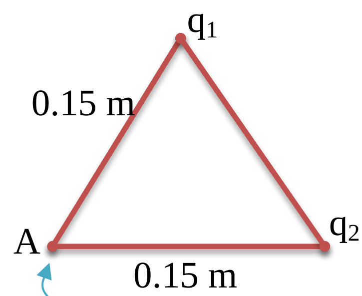
$= 96$ J

Ex:17 Two points charges $q_1 = 15\mu c$, $q_2 = 20\mu c$ are placed at two corners of an equilateral triangle of length 15cm. the amount of work required to bring a charge $q_3 = -5\mu c$ to the third corner of the triangle from infinity is:

a) 10.5 J b) −10.5 J c) 1.05 J d) −1.05 J

Solution

$q_1 = 15 \times 10^{-6}$ C , $q_2 = 20 \times 10^{-6}$ C , $q_3 = -5 \times 10^{-6}$ C



$$V_A = K\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$= 9 \times 10^{9}\left[\frac{15 \times 10^{-6}}{0.15} + \frac{20 \times 10^{-6}}{0.15}\right]$$

$$= 2.1 \times 10^{6} \text{ V}$$

$V_1 = 0$

$V_2 = V_A$

النقل من مالا نهاية

$W = q_3 V$

∞ q_3

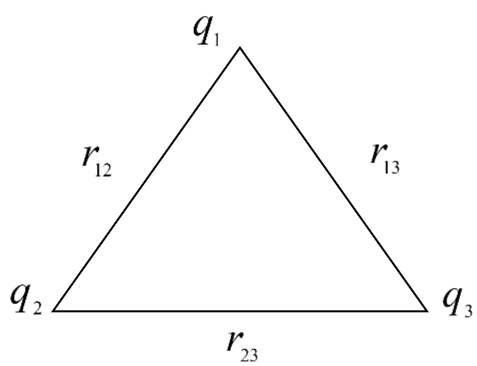
$= -5 \times 10^{-6} \times 2.1 \times 10^{-6}$

$= -10.5$ J

The electric potential energy (U)
طاقة الوضع الكهربية

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

$$\boxed{U = K\frac{q_1 q_2}{r}} \quad J$$



$$U = K\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

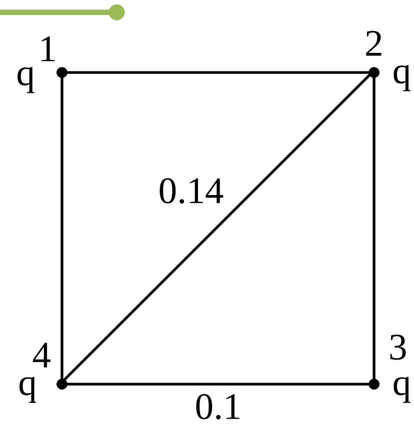
W = U .

وتعتبر هذه الطاقة هي نفسها الشغل اللازم لنقل الشحنات من المالانهاية إلى أماكنها:-

Ex:18 The work is required to bring four identical $+4\mu c$ charges from infinity to the four corners of a square of side 10 cm long is:

a) 8.7 J   b) 12 J   c) 7.8 J   d) 3.4 J

Solution



$$q = 4\times10^{-6} \ C$$

$$q_1 = q_2 = q_3 = q_4 = q$$

$$W = U = K\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_1q_4}{r_{14}} + \frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right]$$

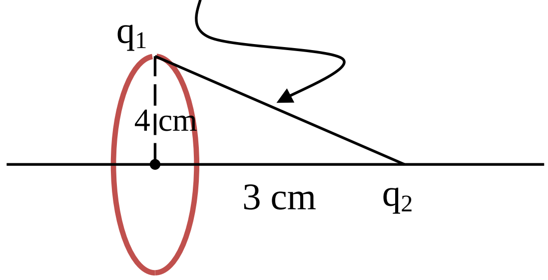
$$= 9\times10^9\times(4\times10^{-6})^2\left[\frac{1}{0.1} + \frac{1}{0.14} + \frac{1}{0.1} + \frac{1}{0.1} + \frac{1}{0.14} + \frac{1}{0.1}\right] = 7.8 \ J$$

Ex:19 A ring of radius R=4cm, with its axis along the x-axis carries a uniformly distributed charge of 8nc. A small particle of charge 5nc is placed at a distance of x = 3cm from the center of the ring. The potential energy of the charge system is:

a) 7.2μJ b) 2.7μJ c) 3.2μJ d) 1.4μJ

Solution

$$r = \sqrt{3^2 + 4^2} = 5 \text{ cm}$$



$$r = 0.05 \text{ cm}$$
$$q_1 = 8 \times 10^{-9} \text{ C}$$
$$q_2 = 5 \times 10^{-9} \text{ C}$$
$$U = K \frac{q_1 q_2}{r}$$
$$= 9 \times 10^9 \frac{8 \times 10^{-9} \times 5 \times 10^{-9}}{0.05}$$
$$= 7.2 \times 10^{-6} \text{ J} = 7.2 \ \mu \text{J}$$

وضع شحنة كهربية في مجال كهربي

عند وضع (تحرك) شحنة كهربية تحت تأثير فرق جهد V مبتدئة من v_0 فإن سرعتها النهائية v.

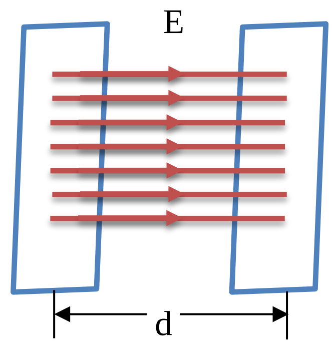
$$|q|V = 0.5m(v^2 - v_0^2)$$

طاقة الحركية ($\Rightarrow$ يتحول إلى) الطاقة الكهربية (الشغل الكهربي)

$$v = \sqrt{\frac{2qV}{m}} + v_0$$

لو السرعة الابتدائية من السكون $V_0 = 0$

$$v = \sqrt{\frac{2qV}{m}}$$



V = E d

فرق الجهد بين اللوحين

شدة المجال الكهربي

البعد بين اللوحين

Ex:20 Two parallel plates are connected to a 100V- power supply. An electron starts from rest at one plate and accelerates toward the other. The speed of the electron when it hits the second plate is:

a) $4\,Mm/s$ b) $6\,Mm/s$ c) $3\,Mm/s$ d) $1.5\,Mm/s$

Solution

$v_0 = 0$

$V = 100\ V$

$q = e = 1.6\times10^{-19}\ C$

$m = 9.1\times10^{-31}\ kg$

$v = ??$

$$v=\sqrt{\frac{2qV}{m}}$$

$$=\sqrt{\frac{2\times1.6\times10^{-19}\times100}{m9.1\times10^{-31}}}$$

$$=5.93\times10^{6}\ m/s$$
$$=6\ \ Mm/s$$

Ex:21 A point charge particle of mass $2\times10^{-6}kg$ and charge $q=1\mu c$ and initially at rest. Falls under a potential difference of $2\times10^{6}V$ its final speed is:

a) $2\,km/s$ b) $1.14\,Km/s$ c) $2.5\,Km/s$ d) $3.4\,Km/s$

Solution

$m = 2\times10^{-6}\ kg$
$V_0 = 0$
$q = 1\times10^{-6}\ C$
$V = 2\times10^{6}\ V$

$$v=\sqrt{\frac{2qV}{m}}=\sqrt{\frac{2\times1\times10^{-6}\times2\times10^{6}}{2\times10^{-6}}}$$

$$=1.14\times10^{3}\ m/s = 1.14\ \ Km/s$$