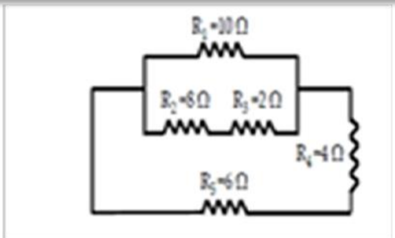
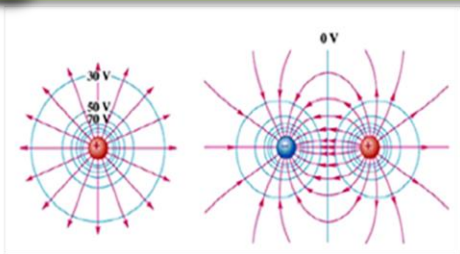
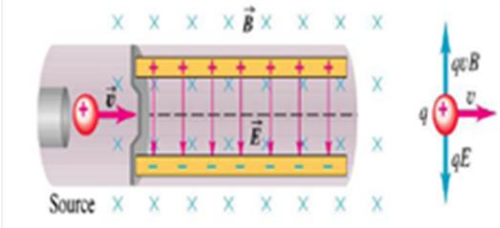
Physics-2







Ch-5

Capacitance

يوسـ زويل ـف

Best wishes and good Luck

00966502047005

Capacitance
السعة الكهربية

ويسمى الخازن الكهربي بالمكثف **Capacitor**

** ولأي مكثف كهربي يكون:

Charge: Q
Capacitance: C
Potential difference: V

$$C = \frac{Q}{V} \qquad V = \frac{Q}{C} \qquad Q = C.V$$

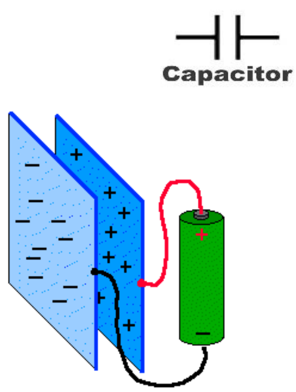
وحدة قياس السعة الكهربية هي الفاراد F وأجزاؤه:-

$nF \xrightarrow{\times 10^{-9}} F$

$\mu F \xrightarrow{\times 10^{-6}} F$

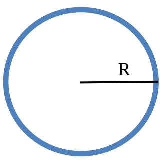
$pF \xrightarrow{\times 10^{-12}} F$

$mF \xrightarrow{\times 10^{-3}} F$



أنواع المكثفات

Isolated Sphere Capacitor



$$C = \frac{R}{K}$$

Parallel Plate Capacitor

مكثف هوائي — **به مادة عازلة**

السعة الكهربية (F)

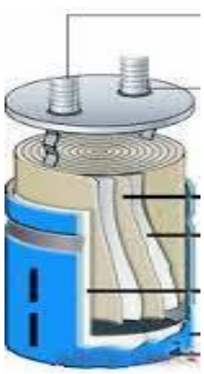
$$\boxed{C = \in_0 K \frac{A}{d}}$$

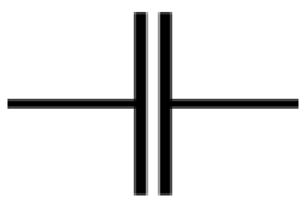
$(8.85 \times 10^{-12})\ C^2/N.m^2$

$K \geq 1$
ثابت العزل
للمادة العازلة

ملحوظة: $k = 1$ للهواء فتكون سعة المكثف الهوائي هي $C = \in_0 \frac{A}{d}$

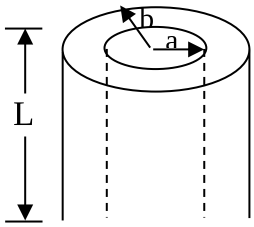
$$\boxed{C = K\,C_0}$$





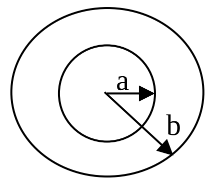
ويوجد من المتوازي نوعان

Cylindrical Capacitor اسطواني



$$C = \frac{1}{2K} \frac{L}{\ln(b/a)}$$

Spherical Capacitor كروي

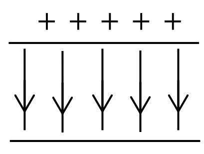


$$C = \frac{1}{K} \frac{ab}{b-a}$$

$$K = 9 \times 10^9 \, N.m^2 / c^2$$

هام جداً:- (للمكثف متوازي اللوحين):-

1) بما أنه لوحين متوازيين:



شدة المجال

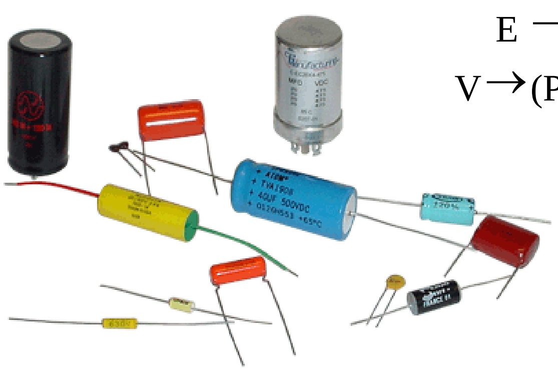
$$E = \frac{V}{d} \quad \text{and} \quad E = \frac{\sigma}{\epsilon_0 K}$$

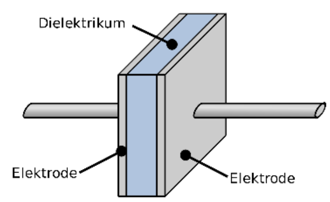
ثابت العزل ويساوي (1) في حالة الهواء

$$\boxed{E = \frac{Q}{\epsilon KA}}$$

E → (Electric field)

V → (Potential difference





2) يختزن المكثف طاقة كهربية U تحسب من العلاقة:-

Stored energy

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{V} \quad J$$

$$\text{kw.h} \xrightarrow{\times 1000 \times 3600} J(w.s)$$

وتحسب كثافة الطاقة μ (energy density) من العلاقة:-

$$\mu = \frac{1}{2} \in_0 E^2 \ (J/m^3)$$

3) للمكثف الواحد الهوائي إذا وضعت به مادة عازلة يكون:

في حالة المادة العازلة

$$C = KC_0$$

في حالة وجود الهواء

$$E = \frac{E_0}{K}$$

في حالة وجود الهواء

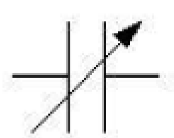
في حالة المادة العازلة

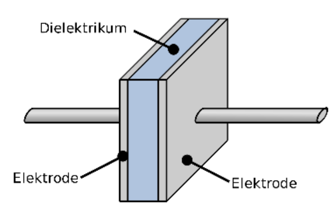
4) في حالة وجود مادة عازلة تتولد عليها شحنة تأثيرية Q' induced:

الشحنة على المادة العازلة
(Induced charge)

$$Q' = Q\left(1 - \frac{1}{K}\right)$$

الشحنة على لوح المكثف





Ex:1 A parallel plate capacitor of plate area $40\,Cm^2$ and separation 2mm is fully charged. If the charge on the capacitor is 9nc, then the potential difference between the plates is:-

a) 250 V b) 120.7 V c) 312.8 V d) 508.5 V

Solution

$$A = 40 \times 10^{-4}\ m^2$$

$$d = 2 \times 10^{-3}\ m$$

$$Q = 9 \times 10^{-9}$$

$$V = ??$$

$$C = \epsilon_0 \frac{A}{d}$$

$$= 8.85 \times 10^{-12} \frac{40 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 1.77 \times 10^{-11}$$

$$V = \frac{Q}{C} = \frac{9 \times 10^{-9}}{1.77 \times 10^{-11}} = 508.5\ V$$

Ex:2 A capacitor stores an energy of 100 kw.h at a potential deference of $10^4 V$. The capacitance of the capacitor is:-

a) 8 F b) 7.2 F c) 2.7 F d) 3.8 F

Solution

$$U = 100 \times 1000 \times 3600\ J$$

$$= 3.6 \times 10^8\ J$$

$$V = 10^4\ V$$

$$C = ??$$

$$U = \frac{1}{2} C V^2$$

$$C = \frac{2U}{V^2}$$

$$= \frac{2 \times 3.6 \times 10^8}{(10^4)^2} = 7.2\ F$$

Ex:3 If a potential deference of 220 V is applied to a parallel plate capacitor of plate area $200\,cm^2$ and separation 2mm.

1) The capacitance of capacitor is:-

a) $6\times10^{-11}F$ b) $4.5\times10^{-11}F$ c) $13.4\times10^{-11}F$ d) $25\times10^{-11}F$

2) The energy stored in the capacitor is:-

a) $3\times10^{-6}J$ b) $2.5\times10^{-6}J$ c) $1.5\times10^{-6}J$ d) $8.14\times10^{-6}J$

Solution

$V = 220\ V$

$A = 200 \times 10^{-4}\ m^2$

$d = 3 \times 10^{-3}\ m$

$C = ??$

$U = ??$

(1)
$$C = \epsilon_0 \frac{A}{d}$$
$$= 8.85 \times 10^{-12} \frac{2 \times 10^{-2}}{3 \times 10^{-3}}$$
$$= 5.9 \times 10^{-11}$$
$$= 6 \times 10^{-11}\ F$$

----

(2)
$$U = \frac{1}{2} C V^2$$
$$= \frac{1}{2} \times 6 \times 10^{-11} \times (220)^2$$
$$= 1.5 \times 10^{-6}\ J$$

Ex:4 A certain air filled parallel-plate capacitor is connected across a 30 V battery. When the battery is disconnected and dielectric slab is inserted into and fills the region between the plates. The voltage across the plates drops by 16 V. the dielectric consultant of slap is:-

a) 3 b) 2.14 c) 2.5 d) 1.8

Solution

وصلت البطارية حتى وصل الجهد إلى 30 V ثم فصلت

وبالتالي تتبقى الشحنة Q كما هي ثابتة

$V_1 = 30 \ V$

$Q_1 = Q_2$

$V_2 = 30 - 16 = 14 \ V$

$K = ??$

$$\because C = C_0 K \quad \left\{ \begin{array}{l} C_0 = \dfrac{Q_1}{V_1} \\ \\ C = \dfrac{Q_2}{V_2} \end{array} \right.$$

$$\frac{Q_2}{V_2} = \frac{Q_1}{V_1} \cdot K$$

$$K = \frac{V_1}{V_2} = \frac{30}{14} = \underline{\underline{2.14}}$$

Ex:5 A parallel plates capacitor of plate separation 1mm has dielectric strength (maximum electric field that it can tolerate) of 20 kv/m the maximum voltage that can be applied to it before it breaks down is:-

a) 17.5V   b) 13.4V   c) 20V   d) 19.5V

Solution

$$d = 1 \times 10^{-3} \text{ m}$$

$$V = ??$$

$$E = 20 \times 10^{3}$$

$$E = \frac{V}{d}$$

$$V = E \cdot d$$

$$= 20 \times 10^{3} \times 1 \times 10^{-3}$$

$$= 20 \text{ V}$$

Ex:6 The plates of spherical capacitor have radii 38mm and 40 mm.

1) The capacitance of this spherical capacitor is:

a) 2 PF b) 0.84 PF c) 0.3 PF d) 1.5 PF

2) The plate area of a parallel-plat capacitor with the same plate separation and capacitance is:-

a) $1\ m^2$ b) $0.3\ m^2$ c) $0.02\ m^2$ d) $1.2\ m^2$

Solution

(1)

$$a = 38 \times 10^{-3}\ m$$

$$b = 40 \times 10^{-3}\ m$$

$$C = ??$$

$$C = \frac{1}{K}\ \frac{a\,b}{b - a}$$

$$= \frac{1}{9 \times 10^{9}}\ \frac{40 \times 38 \times 10^{-6}}{2 \times 10^{-3}}$$

$$= 8.4 \times 10^{-11}\ F$$

$$= 0.84\ PF$$

(2)

$$d = b - a$$

$$= 2 \times 10^{-3}$$

$$C = 8.4 \times 10^{-11}$$

$$A = ??$$

$$C = \epsilon_0\ \frac{A}{d}$$

$$A = \frac{C\,d}{\epsilon_0}$$

$$= \frac{8.4 \times 10^{-11} \times 2 \times 10^{-3}}{8.85 \times 10^{-12}} = 0.02\ m^2$$

Ex: 7 A parallel plate capacitor of plate area $4\times10^{-3}m^2$ and capacitance 36 PF. Is charged to a potential difference of 500 V the energy density between the plate's capacitor is:-

a) $2.5J/m^3$ b) $13J/m^3$ c) $1.14J/m^3$ d) $1.5J/m^3$

Solution

$A = 4\times10^{-3}\ m^2$

$C = 36\times10^{-12}\ F$

$V = 500\ V$

$U = ??$

طريقة اخرى

(1) $C = \dfrac{Q}{V}$ $Q = ??$

(2) $E = \dfrac{Q}{\epsilon_0 A}$

(3) $U = \dfrac{1}{2}\epsilon_0 E^2$

$* C = \epsilon_0 \dfrac{A}{d}$

$d = \dfrac{\epsilon_0 A}{C} = \dfrac{8.4\times10^{-11}\times 2\times10^{-3}}{8.85\times10^{-12}}$

$= 0.98\times10^{-4}\ m$

$* E = \dfrac{V}{d} = \dfrac{500}{9.8\times10^{-4}}$

$= 5.08\times10^{5}\ V/m$

$* U = \dfrac{1}{2}\epsilon_0 E^2$

$= \dfrac{1}{2}\times 8.85\times10^{-12}\times(5.08\times10^5)^2$

$= 1.14\ J/m^3$

Ex:8 Two parallel plate of area 100cm^2 are given charges of equal magnitude $9\times10^{-17}\text{C}$ but opposite signs. The electric field within the dielectric material filling the space between the plates is $1.5\times10^6\,\text{V/m}$

1) The dielectric constant is:-

a) 6.8 b) 4.2 c) 8.6 d) 1.22

2) The magnitude of the charge induced on:-

a) 0.77μc b) 0.46μc c) 0.89μc d) 1.2μc

Solution

$A = 100\times10^{-4}\ \text{m}^2$ $\quad Q = 9\times10^{-7}\ \text{C}$ $\quad E = 1.5\times10^{6}\ \text{V/m}$

$K = ??$ $\quad Q' = ??$

$$E = \frac{Q}{\varepsilon_0\ K\ A} \gg$$

(1) $K = \frac{Q}{\varepsilon_0\, A\, E}$

$$K = \frac{9\times10^{-7}}{1.5\times10^{6}\times8.8\times10^{-12}\times10^{-2}} = 6.8$$

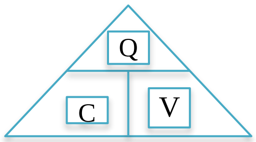
(2) $$Q' = Q\left(1-\frac{1}{K}\right)$$

$$= 9\times10^{-7}\left(1-\frac{1}{6.8}\right)$$

$$= 7.7\times10^{-7}\ \text{C} = 0.77\ \mu\text{C}$$

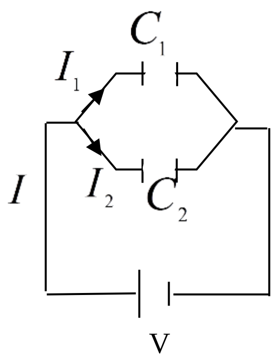
Capacitors connection

توصيل المكثفات



للحصول على سعة اكبر وجهد اقل

Parallel connection



الجهد ثابت في هذا النوع من التوصيل

$$V = V_1 = V_2 = \ldots\ldots$$

<u>لكن</u>

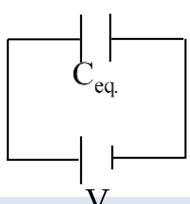
$$Q = Q_1 + Q_2$$

$$C_{eq} = C_1 + C_2$$

$$Q_1 = C_1.V$$

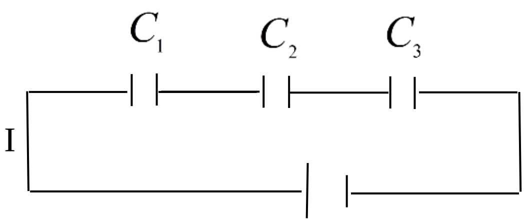
$$Q_2 = C_2.V$$

$$Q_1 + Q_2 = C_{eq}.V$$



للحصول على سعة اقل وجهد اكبر

Series connection



الشحنة ثابتة في هذا النوع من التوصيل

$$Q_1 = Q_1 = Q_2 = C_{eq}.V$$

<u>لكن</u>

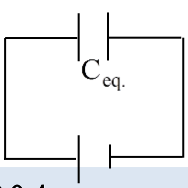
$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C_{ey}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

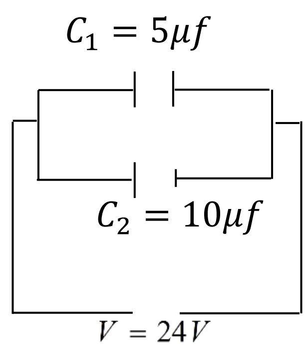
$$V_1 = \frac{Q_1}{C_1} \quad , \quad V_2 = \frac{Q_2}{C_2} \quad , \quad V_3 = \frac{Q_3}{C_3}$$

* لعدد مكثفين فقط على التوالي:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



Ex: 9 in the figure



1) The equivalent capacitance is:-

a) 2μF b) 10μF c) 15μF d) 5μF

2) The voltage (potential difference) across each capacitor is:-

a) 5 V b) 10 V c) 24 V d) 30 V

3) The charge on each capacitor is:-

a) 120μc,240μc b) 240μc,120μc

c) 80μc,160μc d) 160μc,80μc

Solution

$C_1 = 5 \times 10^{-6}$ F $\quad C_2 = 10 \times 10^{-6}$ F $\quad V = 24$ V

(1) $C_1 // C_2 \rightarrow C_{eq} = C_1 + C_2 = 5 \times 10^{-6} + 10 \times 10^{-6} = 15 \times 10^{-6}$ F

----

(2) $C_1 // C_2 \rightarrow V_1 = V_2 = V = 24$ V

----

(3) $C_1 // C_2 \rightarrow Q_1 = C_1 \cdot V$

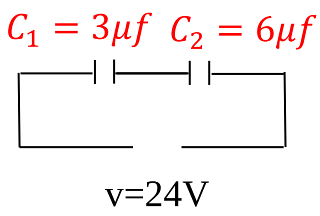
$= 5 \times 10^{-6} \times 24 = 1.2 \times 10^{-4}$ C $= 120$ μc

$Q_2 = C_2 \cdot V$

$= 10 \times 10^{-6} \times 24 = 2.4 \times 10^{-4}$ C $= 240$ μc

Ex: 10 in the figure

1) The equivalent capacitance is:-



a) 3μF b) 6μF c) 2μF d) 9μF

2) The charge on each capacitor is:-

a) 12μc b) 48μc c) 4μc d) 3μc

3) The voltage across each capacitor is:-

a) 4V,2V b) 16V,8V c) 5V,10V d) 9V,18V

Solution

$C_1 = 3 \times 10^{-6}$ F $\quad C_2 = 6 \times 10^{-6}$ F $\quad V = 24$ V

(1) $C_1, C_2 \xrightarrow[(2)]{\text{series}} C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{3\times6\times10^{-12}}{10^{-6}(3+6)} = 2 \times 10^{-6}$ F

Or $\dfrac{1}{C_{eq}} = \dfrac{1}{3} + \dfrac{1}{6} = \dfrac{2+1}{6} = \dfrac{3}{6} = \dfrac{1}{2}$

$\dfrac{1}{C_{eq}} = \dfrac{1}{2} \xRightarrow{\text{تقلب}} C_{eq} = 2\ \mu\text{F} = 2 \times 10^{-6}$ F

---

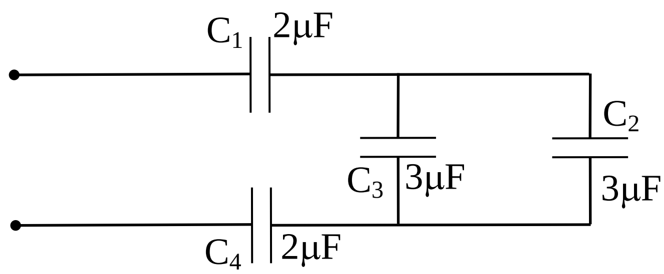
(2) $C_1, C_2 \xrightarrow{\text{series}} Q_1 = Q_2 = C_{eq} \cdot V$

$= 2 \times 10^{-6} \times 24 = 48 \times 10^{-6}$ C $= 48$ μc

---

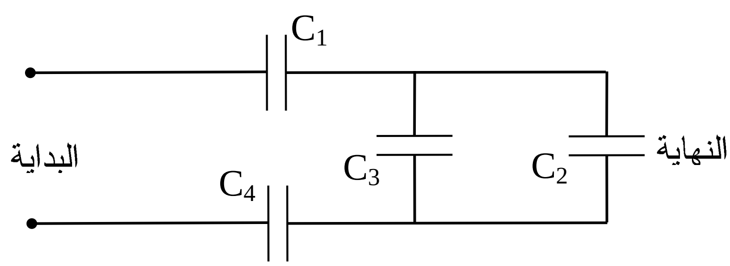
(3) $V_1 = \dfrac{Q}{C_1} = \dfrac{48\times10^{-6}}{3 \times 10^{-6}} = 16$ V & $V_2 = \dfrac{Q}{C_2} = \dfrac{48\times10^{-6}}{6 \times 10^{-6}} = 8$ V

Ex: 11 in the figure shown the equivalent capacitance of the combination of capacitors is:



a) 2μF b) 1μF c) 0.86μF d) 1.3μF

Solution



عند حساب C_{eq} لمثل هذه الدوائر نبدأ دائماً من نهاية الدائرة

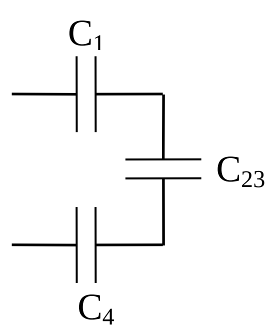
بحساب C_{eq} لكل توصيلة متشابهة تدريجياً حتى نصل إلى البداية

* $C_2 // C_3 \rightarrow C_{23} = C_2 + C_3 = 3 + 3 = 6$ μf

----

* $C_1, C_{23}, C_4 \xrightarrow{\text{series}} \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}$

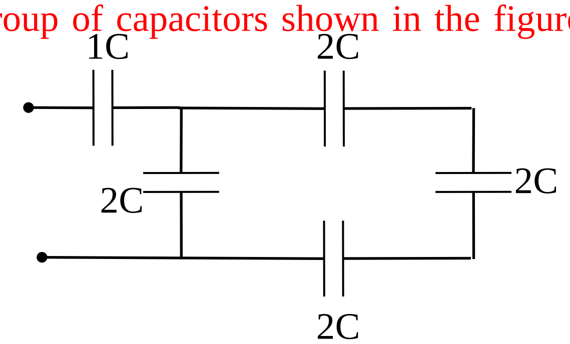
$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{7}{6}$$



$$\therefore \; C_{eq} = \frac{6}{7} = 0.86 \; \mu f$$

Ex:12 If $V_{ab} = 30V$. the energy stored by the group of capacitors shown in the figure is:-

$C = 24\mu F$



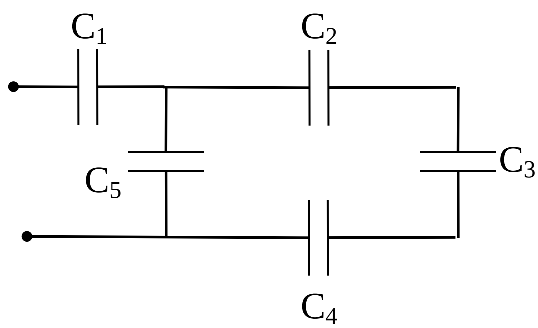
a) 2.5mJ b) 4mJ c) 5.1mJ d) 3.5mJ

Solution

$C_1 = 24\mu f$ $V = 30 \; V$

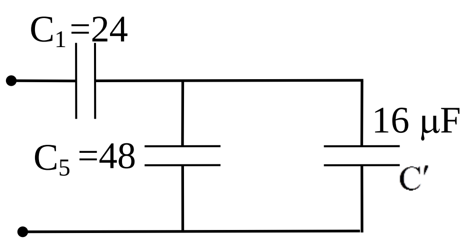
$C_2 = C_3 = C_4 = C_5 = 48 \; \mu f$

$U = ??$ $U = \frac{1}{2} C_{eq} \cdot V^2$



$$C_2 \;,\; C_3 \;,\; C_4 \xrightarrow{\text{series}} \frac{1}{C'} = \frac{1}{48} + \frac{1}{48} + \frac{1}{48} = \frac{3}{48} \Rightarrow C' = \frac{48}{3} = 16 \; \mu f$$

* $C_5 \,//\, C' \;\rightarrow\; C'' = C_5 + C = 48 + 16 = 64 \; \mu f$



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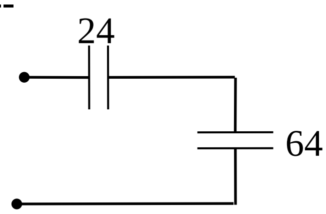
* $C_1 \;,\; C'' \xrightarrow{\text{series}} C_{eq} = \frac{24 \times 64}{24 + 64} = 17.5 \; \mu f$



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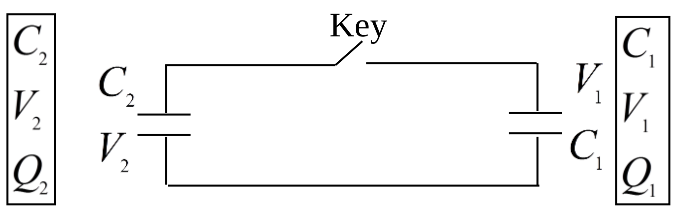
* $U = 17.5 \times 10^{-6} \times (30)^2 \times \frac{1}{2} = 7.9 \times 10^{-3} \; J = 7.9 \; mJ$

هام هام جداً:-



إذا كان لدينا المكثفان
ووصلناهما كما بالشكل

فإن الشحنة تنتقل بينهما ويصلا للتوازن ويكون

$$V_1 = V_2 \Rightarrow \quad \frac{Q'_1}{C_1} = \frac{Q'_2}{C_2}$$

$$Q_1 + Q_2 = Q'_1 + Q'_2$$

مشابهة لتوصيل كرتين مشحونتين التى سبق دراستها

$$V_{Com} = \frac{Q'_1}{C_1} = \frac{Q'_2}{C_2}$$

Ex:13 A $30\mu F$ capacitor charged to 3 V and $40\mu F$ capacitor charged to 4 V are connected to each other $(+,+)(-,-)$ the charge on the $40\,\mu F$ capacitor after the two capacitors are connected and reached equilibrium is:-

a) $2\times10^{-5}C$ b) $1.4\times10^{-4}C$ c) $3.5\times10^{-11}C$ d) $3.8\times10^{-4}C$

Solution

$C_1 = 30\times10^{-6}$ $V_1 = 3\text{ V}$ $Q'_2 = ??$

$C_2 = 40\times10^{-6}$ $V_2 = 4\text{ V}$

$\Rightarrow\ Q_1 = C_1V_1 = 90\times10^{-6}\text{ C}\ \ \&\ \ Q_2 = C_2V_2 = 160\times10^{-6}\text{ C}$

$\Rightarrow\ Q_1 + Q2 = Q'_1 + Q'_2 = 250\times10^{-6}$

$$Q'_1 = (250\times10^{-6} - Q'_2)$$

$$\Rightarrow\ \frac{Q'_1}{C_1} = \frac{Q'_2}{C_2}$$

$$\therefore\ \frac{250\times10^{-6} - Q'_2}{30\times10^{-6}} = \frac{Q'_2}{40\times10^{-6}}$$

$$\therefore\ 30\times10^{-6}\cdot Q'_2 = 10^{-8} - 40\times10^{-6}\cdot Q'_2$$

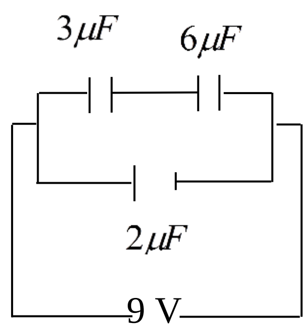
$$\therefore\ 70\times10^{-6}\cdot Q'_2 = 10^{-8}\ \Rightarrow\ Q'_2 = 1.4\times10^{-4}\text{ C}$$

ملحوظة: إذا طلب الجهد المشترك لهما بعد الاتزان يكون

$$V = \frac{Q'_2}{C_2} = \frac{1.4\times10^{-4}}{40\times10^{-6}} = 3.5\ \text{V}$$

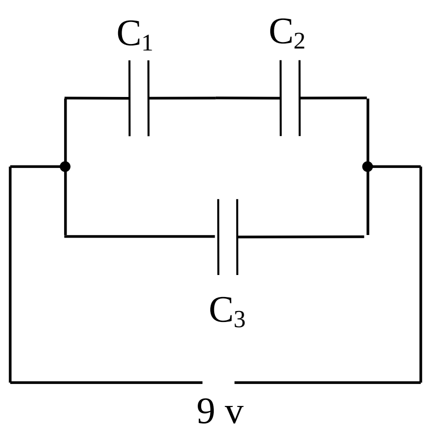
Ex: 14 in the figure shown the energy stored

In the $3\mu F$ Capacitor is:



a) $18\mu C$ b) $27\mu J$ c) $54\mu J$ d) $9\mu J$

Solution



C_1 & C_2

$Q_1 = Q_2$

$$\boxed{V_1 + V_2 = 9}$$

لأنهما // مع البطارية
وعلى التوالي معاً

$C_1 V_1 = C_2 V_2$

$C_1 V_1 = C_2 (9 - V_1)$

$C_1 V_1 = 9 C_2 - V_1 C_2$

$3\times10^{-6}\ V_1 = 9 \times 6\times10^{-6} - 6\times10^{-6}\ V_1$

$9\times10^{-6}\ V_1 = 9\times 6\times10^{-6}$

$$\therefore \boxed{V_1 = 6\ \text{V}}$$

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}\times3\times10^{-6}\times(6)^2$$

$$= 54\times10^{-6}\ \text{J} = 54\mu J$$