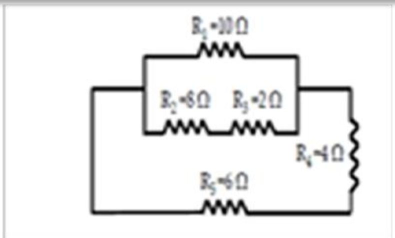
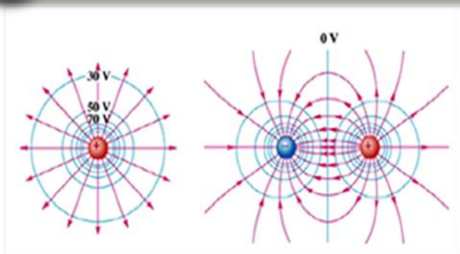
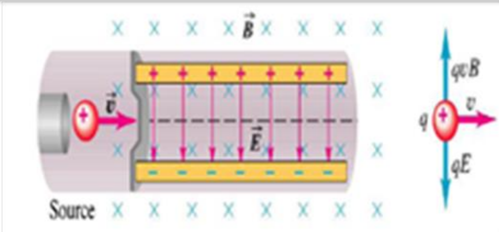
Physics-2







Ch-3

Gauss Law

يوســ زويل ــف

Best wishes and good Luck

00966502047005

Gauss Law

قانون جاوس

$$\phi = \int \vec{E} . d\vec{A}$$

$$\in_0 \ \phi = q$$

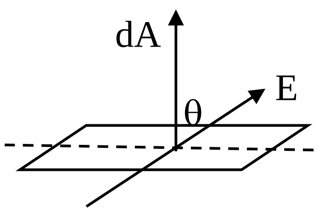
1) The Electric Flux

الفيض (التدفق الكهربي خلال سطح)

Gauss Law

إذا كان السطح موضوع داخل مجال كهربي منتظم $\phi = \int \vec{E} . d\vec{A}$

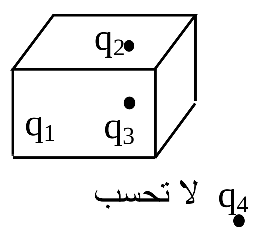
$\varphi = E A Cos\theta$



إذا كان السطح مغلق على شحنة داخلة

$$\in_0 \ \phi = q_{enc}$$

$$\phi = \frac{q}{\in_0}$$



q_4 لا تحسب

$\phi \rightarrow$ Flux الفيض

$\vec{E}$ Electric field

$\overrightarrow{dA}$ متجه مساحة السطح

$q_{enc} \rightarrow$ الشحنة التي داخل السطح

θ الزاوية بين اتجاه $\overrightarrow{dA}$ و $\vec{E}$

Ex1: Two charges 15 pc, -40 pc are placed inside a cube of edge Length 40 cm. the net electric flux through the surface of the cube is:

a) $5\ N.m^2/c$ b) $2.8\ N.m^2/c$

c) $-2.8\ N.m^2/c$ d) $28\ N.m^2/c$

Solution

$q_1 = 15 \times 10^{-12}\ c$

$q_2 = -40 \times 10^{-12}\ c$

$L = 0.4\ m$

$q_{enc} = q_1 + q_2$

$= 15 \times 10^{-12} + (-40 \times 10^{-12})$

$= -25 \times 10^{-12}$

$$\phi = \frac{q_{enc}}{\epsilon_0} = \frac{-25 \times 10^{-12}}{8.85 \times 10^{-12}} = -2.8\ N.m^2/c$$

Ex:2 A charge of 4nc is placed at the center of a cube that measures 0.4 m on each side. The electric flux through one face of the cube is:

a) $4.25\ N.m^2/c$ b) $75.3\ N.m^2/c$

c) $7.53\ N.m^2/c$ d) $13.5\ N.m^2/c$

Solutio

$q_{enc} = 4 \times 10^{-9}\ c$

المطلوب حساب التدفق (الفيض) على وجه واحد للمكعب

$$\phi_{total} = \frac{q_{enc}}{\epsilon_0} = \frac{4\times10^{-9}}{8.85\times10^{-12}} = = 451.98\ N.m^2/c$$

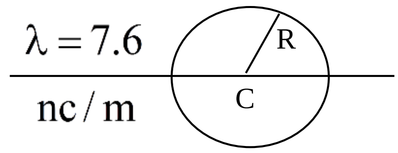
وللوجه الواحد نقسم عل 6

$$\phi_{0f\ one\ face} = \frac{\phi_{total}}{6} = \frac{452}{6} = 75.3\ N.m^2/c$$

Ex:3 An infinitely long line has a charge density of 7.6 nC/m. The electric flux through a spherical surface of radius R=8 cm whose c enter, c, lies on the line of charge as shown in the figure.

a) $13.74\ N.m^2/c$　　b) $137.4\ N.m^2/c$

c) $22\ N.m^2/c$　　d) $130\ N.m^2/c$



Solutio

$\lambda = 7.6\ nc/m$

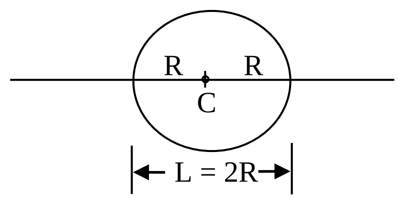
$R = 0.08\ m$

$L = 2R$

نصف قطر

السطح الكروي يحوي داخله جزء من الخط المشحون طوله 2R

$$q_{enc} = \lambda \cdot L = \lambda \cdot 2R = 7.6\times10^{-9}\times 2\times 0.08 = 1.216\times10^{-9}\ c$$



$$\phi = \frac{q_{enc}}{\epsilon_0} = \frac{1.23\times10^{-9}}{8.85\times10^{-12}} = 137.4\ \ N.m^2/c$$

Ex:4 When a piece of paper is held with one face perpendicular to a uniform electric field, the electric flux was $48 N.m^2/c$. So when the plane of the paper makes 30° with the direction of the electric field the electric flux is:

a) $22\ N.m^2/c$ b) $13.5\ N.m^2/c$

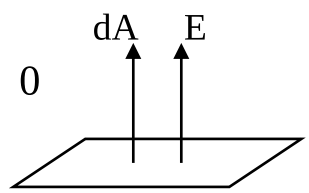
c) $12.4\ N.m^2/c$ d) $24\ N.m^2/c$

Solutio

$\phi_1 = 48$ $\theta_1 = 0$

$\phi_2 = ??$ $\theta = 60$

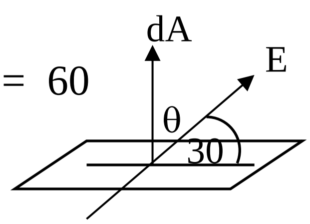
$\theta_1 = 0$



$$\frac{\phi_1}{\phi_2} = \frac{E\,A\cos\theta_1}{E\,A\cos\theta_2} = \frac{\cos\theta_1}{\cos\theta_2}$$

$$\frac{48}{\phi_2} = \frac{\cos 0}{\cos 60}$$

$\theta_2 = 60$



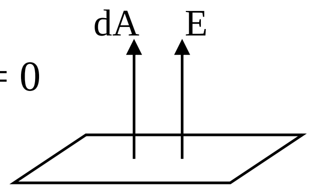
$$\therefore \frac{48}{\phi_2} = \frac{1}{0.5}$$

$$\phi_2 = 48 \times 0.5 = 24\ N.m^2/c$$

Ex:5 If the electric field is perpendicular on the square of side of 5 cm length and E = 5 KN/C the net charge on the square is:

a) 1.1×10^{10}C b) 1.1×10^{10}C c) 2.5×10^{-10}C d) 3.4×10^{-10}C

Solutio



$L = 0.05$ m $\quad$ $A = 25\times10^{-4}$ m^2

$E = 5000$ N/C $\quad$ $\theta = 0$

$\phi = EA\cos\vartheta = \frac{q_{enc.}}{\varepsilon_0}$ $\quad$ Cos 0=1

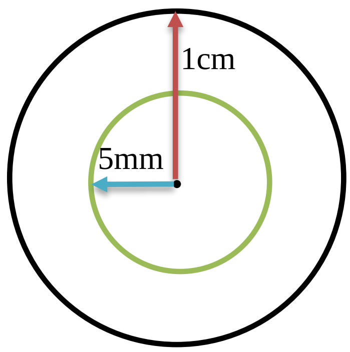
$$q_{enc} = EA\epsilon_0\cos\theta = 5000\times25\times10^{-4}\times8.85\times10^{-12}\times1$$
$$= 1.1\times10^{-10}\ \text{C}$$

Ex:6 An insulating sphere of radius 1cm has uniform charge density $6\times10^{-3} c/m^3$. The electric flux through a concentric spherical surface with R = 5mm is:

a) $300\ N.m^2/c$ b) $355\ N.m^2/c$

c) $351\ N.m^2/c$ d) $215\ N.m^2/c$

Solutio



ملحوظة: الكثافة الحجمية ρ للشحنة ثابتة بالنسبة لكل من الكرتين.

وبالتالي فإن الشحنة الداخلية للكرة الصغرى هي:

$$q_{enc} = \rho \cdot V_{\text{الصغرى}} = \rho \cdot \frac{4}{3}\pi R^3$$

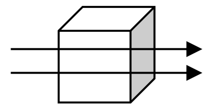
$$= 6\times10^{-3} \times \frac{4}{3} \times 3.14 \times (5\times10^{-3})^3$$

$$= 3.14\times 10^{-9}\ C$$

$$\phi = \frac{q_{enc}}{\epsilon_0} = \frac{3.14\times10^{-9}}{8.85\times10^{-12}} = 355\ N.m^2/c$$

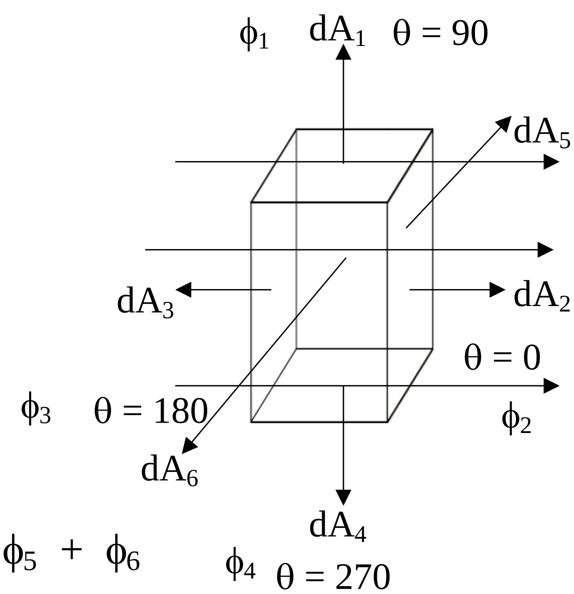
Ex:7 The electric flux through the closed surface as shown is:

a) 3E b) 2EA c) 0 d) $\frac{1}{2}$EA



Solutio

ملحوظة: إذا وضع سطح منتظم مغلق { مكعب – اسطوانة – متوازي مستطيلات}
داخل مجال مغناطيسي منتظم E فإن ϕ الكلية تساوي صفر لهذا السطح.



$\phi_1 = EA\cos 90 = 0$

وكذلك ϕ_4 , ϕ_5 , ϕ_6

$\phi_2 = EA\cos 0 = EA$

$\phi_3 = EA\cos 180 = -EA$

$$\phi_{total} = \Sigma\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$$
$$= 0 + EA + (-EA) + 0 + 0 + 0$$
$$= 0$$

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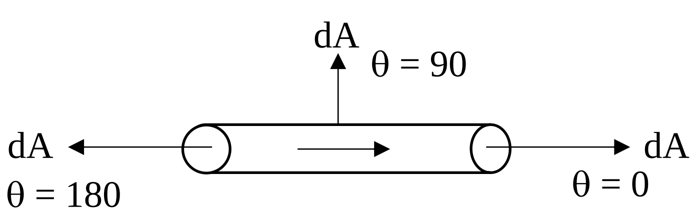
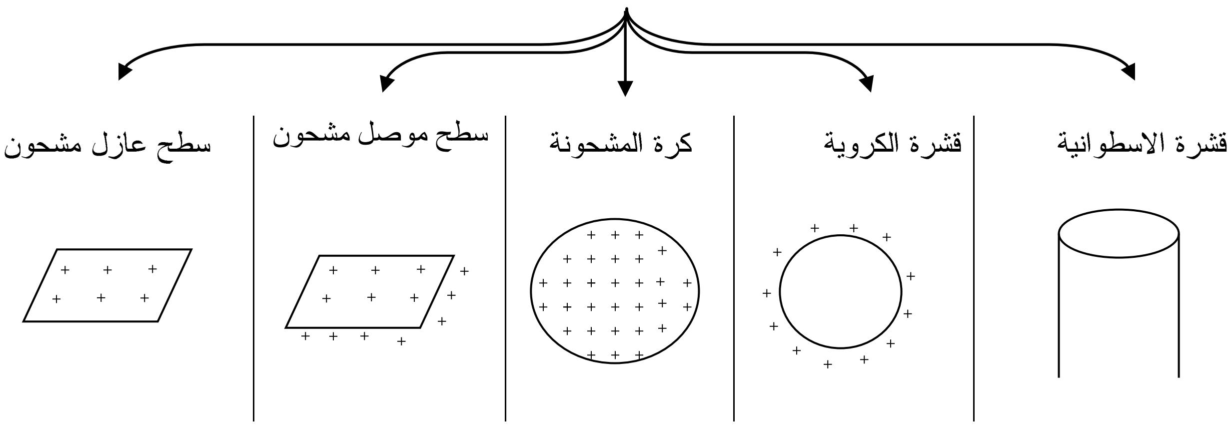
في حالة الاسطوانة أيضاً $\phi_{net} = 0$



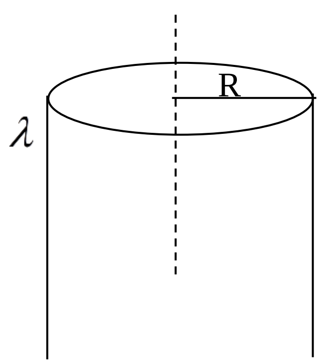
(2) Electric Fields at a points on a Gaussian surface

حساب شدة المجال الكهربي عند نقاط محددة لسطوح جاوس



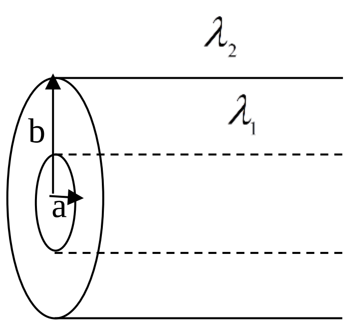
(1) Cylindrical shell

One cylindrical shell



$$E = \begin{cases} 0 & r < R \\ 2k\dfrac{\lambda}{r} & r \geq R \end{cases}$$

Two concentric cylindrical shells



$$E = \begin{cases} 0 & r < a \\ 2k\dfrac{|\lambda_1|}{r} & a \leq r < b \\ 2k\dfrac{|\lambda_1 + \lambda_2|}{r} & r \geq b \end{cases}$$

Ex: 8 Two cylindrical shells are concentric and

$\lambda_1 = 4\mu c/m \quad r_1 = 5cm, \ \lambda_2 = -1\mu c/m, \ r_2 = 10cm$

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(1) The electric field at r = 3 cm is:

a) $20 N/C$ b) 0 c) $1.4 N/C$ d) $3.2 N/C$

---

(2) The electric field at r = 7 cm is:

a) $1\times10^6 N/C$ b) $2.5\times10^6 N/C$

c) $3.1\times10^6 N/C$ d) $4.5\times10^{-6} N/C$

---

(3) The electric field at r = 15 cm is:

a) $2.1\times10^4 N/C$ b) $3.6\times10^5 N/C$

c) $1.1\times10^5 N/C$ d) $2.8\times10^4 N/C$

Solutio

(1) $r = 3\text{ cm} < r_1 \Rightarrow E = 0$

(2) $r = 7\text{ cm} \qquad r_1 < r < r_2$

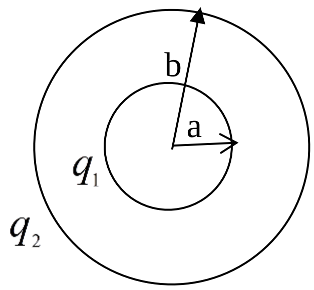
$$E = 2K\frac{|\lambda_1|}{r} = 2\times9\times10^9\ \frac{4\times10^{-6}}{0.07} = 1.03\times10^6\ N/C$$

(3) $r = 15\text{ cm} > r_2$

$$E = 2K\frac{|\lambda_1+\lambda_2|}{r} = 2\times9\times10^9\ \frac{|4\times10^{-6}-1\times10^{-6}|}{0.15} = 3.6\times10^5\ N/C$$

(2) Spherical Shell
القشرة الكروية

Two concentric spherical shells

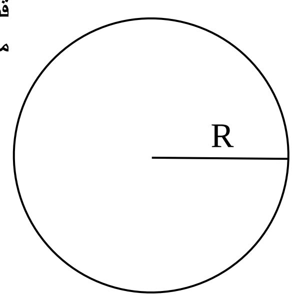


$$E = \begin{cases} 0 & r < a \\ K\dfrac{|q_1|}{r^2} & a \le r < b \\ k\dfrac{|q_1+q_2|}{r^2} & r \ge b \end{cases}$$

$$q = \sigma . A$$
$$= \sigma . 4\pi R^2$$

One spherical shell

قشرة كروية من
مادة عازلة



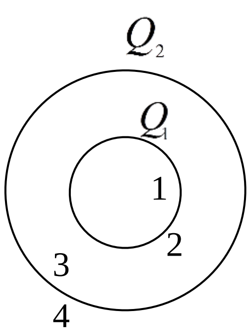
$$E = \begin{cases} 0 & r < R \\ K\dfrac{|q|}{r^2} & r \ge R \end{cases}$$

* لكن الكرة الموصلة (هام)
$E = 0$ على السطح $r = R$

$r \rightarrow$ هي نصف قطر القشرة الكروية المراد حساب E عليها

$R \rightarrow$ نصف قطر القشرة الكروية المشحونة المعطاة

An excess charge on an isolated conductor is located entirely
on the outer surface of the conductor



توزيع الشحنات:-

على سطحي كل shell

1) $Q = 0$
2) $Q = Q_1$
3) $Q = -Q_1$
4) $Q = Q_2 + Q_1$

الجمع جبري أي كل شحنة بإشارتها

Ex:9 A spherical conducting shell of radius 30 cm has surface charge density $8\times10^{-6} c/m^2$. The magnitude of the electric field at a point 0.4m from the surface of the shell is:

a) $1.1\times10^{5} N/C$ b) $1.7\times10^{5} N/C$

c) $2.1\times10^{5} N/C$ d) $3.1\times10^{5} N/C$

Solutio

$R = 30$ cm

$= 0.3$ m

$r = 0.3 + 0.4 = 0.7$ m

$\sigma = 8\times10^{-6}$ c/m^2

$q = \sigma \cdot A$

$= \sigma \cdot 4\pi R^2$

$= 8\times10^{-6} \times 4\times3.14\times0.3^2$

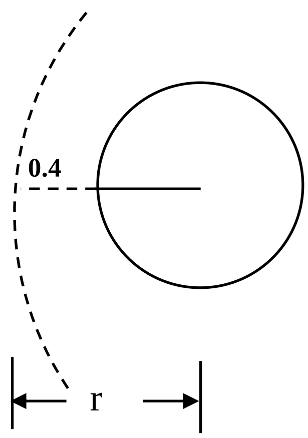
$= 9\times10^{-6}$ C

$r = 0.7 > R$

$$E = K\frac{|q|}{r^2}$$

$$= 9\times10^{9}\ \frac{9\times10^{-6}}{(0.7)^2}$$

$$= 1.7\times10^{5}\ N/C$$



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ملحوظة: في هذا المثال (القشرة موصلة) إذا طلب E عند r = 0.3m تكون r = 0.3 = R

وتكون E = 0 على السطح للقشرة المعدنية (الموصلة)

Ex:10 The electric field just outside the surface of conducting sphere of radius 80cm is 950N/C and points radically toward the center of the sphere. The surface charge density is:

a) $2.4\times10^{-9}c/m^2$

b) $8.4\times10^{-9}c/m^2$

c) $2.1\times10^{3}c/m^2$

d) $8.4\times10^{9}c/m^2$

Solutio

$E = 950$ N/C

$R = r = 0.8$ m

$A = 4\pi R^2$

$$E = K\frac{|q|}{r^2}$$

$$q = \frac{E\cdot r^2}{K}$$

$$= \frac{950\times0.8^2}{9\times10^9} = 6.76\times10^{-8}\ \text{C}$$

وحيث أن اتجاه المجال يشير إلى مركز السطح الكروي

$\therefore$ تكون الشحنة سالبة (−) $q = -6.76\times C^{-8}10$

$q = -6.76\times C^{-8}10$

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2}$$

$$= \frac{-6.76 \times 10^{-8}}{4 \times 3.14 \times 0.8^2} = 8.4 \times 10^{-9} \ \ C/m^2$$

****

Ex:11 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a net charge of $3\mu c$. If a point charge of $-5\mu c$ is placed at the center of the spherical shell. The charge on the outer surface of the spherical shell is:

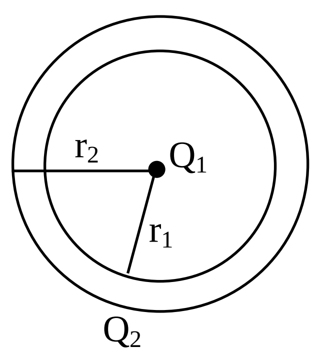
a) $2\mu c$ b) 0 c) $4\mu c$ d) $-2\mu c$

Solutio

توزيع الشحنات

$Q_1 = -5 \times 10^{-6}$ C

$Q_2 = 3 \times 10^{-6}$ C



For the outer surface is

$$Q = Q_1 + Q_2$$

$$= -5 \times 10^{-6} + 3 \times 10^{-6}$$

$$= -2 \times 10^{-6} \ C$$

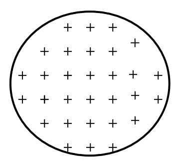
$$= -2 \ \mu C$$

(3) Charged sphere الكرة المشحونة

وهي كرة من مادة عازلة والشحنة موزعة بانتظام على حجمها.

A sphere has a charge uniformly distributed through its volume.

$$E = \begin{cases} K\dfrac{|Q|r}{R^3} & r < R \\ K\dfrac{|Q|}{r^2} & r \geq R \end{cases}$$



$R \leftarrow$ نصف قطر الكرة المشحونة (المعطاة).

$r \leftarrow$ نصف قطر السطح الكروي المراد حساب E عليه.

ملحوظة هامة:-

*(1)

الشحنة التي على السطح الكروي $Q = \rho \cdot \overbrace{\frac{4}{3}\pi r^3}^{\text{حجم الكرة}}$

ρ : الكثافة الحجمية

r : نصف القطر

$$\frac{Q_{surface}}{Q_{in}} = \frac{R^3}{r^3}$$

Q ← هي الشحنة داخل الكرة التي نصف قطرها R.

Q_1 ← هي الشحنة داخل الكرة التي نصف قطرها r. حيث $r < R$

----

r = R على السطح →

$$\frac{E_{surface}}{E_{In}} = \frac{R}{r} \qquad\qquad \frac{E_{out}}{E_{In}} = \frac{R^3}{r_{out}^2 . r_{in}} \qquad (2)^*$$

r < R في الداخل →

Ex:12 An insulating sphere of 12 cm radius has uniform volume charge density of $20nc/m^3$. The magnitude of the electric field at 15cm from the center of the sphere is:

a) $32.4 N/C$ b) $39.6 N/C$ c) $57.6 N/C$ d) $56.7 N/C$

Solutio

$R = 0.12$ m

$\rho = 20 \times 10^{-9}$ c/m^3

$r = 0.15$ m $> R$

$$Q = \rho \cdot \frac{4}{3}\pi R^3$$

$$= 20 \times \frac{4}{3} \times 3.14 \times (0.12)^3 \times 10^{-9}$$

$$= 1.44 \times 10^{-10} \text{ C}$$

$\because r > R$ $\qquad \therefore E = K\frac{|Q|}{r^2}$

$$= 9 \times 10^9 \frac{1.44 \times 10^{-10}}{0.15^2} = 57.6 \text{ N/C}$$

****

Ex:13 An insulating sphere of radius 15cm has a net charge Q distributed uniformly throughout its volume. If the electric field inside the sphere at r = 7 cm is 4000 N/C. the electric field outside the sphere are r = 17 cm is:

a) $6.7KN/C$ b) $8.4KN/C$ c) $7.6KN/C$ d) $0.76kN/C$

Solutio

$R = 0.15\ m$

$r_{1_{in}} = 0.07\ m$

$r_{1_{out}} = 0.17\ m$

$E_{in} = 4000\ N/C$

$E_{out} = ??$

$$\frac{E_{in}}{E_{out}} = \frac{r_2^2\ r_1}{R^3}$$

$$\frac{4000}{E_{out}} = \frac{(0.17)^2 \times 0.07}{(0.15)^3}$$

$$\therefore E_{out} = \frac{4000 \times (0.15)^3}{(0.17)^2 \times 0.07} = 6673.3N/C = 6.7\ KN/C$$

Ex:14 A solid insulating sphere has a charge of $18\mu c$ uniformly distributed throughout its volume. The magnitude of the electric field inside the sphere at r = 4cm and outside the sphere at r = 15cm measured from the center of the sphere are equal .The volume charge density of the sphere is:

a) $9mc/m^3$ b) $4.8mc/m^3$

c) $13.5mc/m^3$ d) $2.5mc/m^3$

Solutio

$r_{1_{in}} = 0.04\ m$

$r_{1_{out}} = 0.15\ m$

$Q = 18 \times 10^{-6}\ C$

$R = ??$

$\rho = ??$

$$E_{in} = E_{out}$$

$$K\frac{|Q|r_1}{R^3} = K\frac{|Q|}{r_2^2}$$

$$R^3 = r_2^2\ r_1$$

$$= 0.15^2 \times 0.04$$

$$= 9 \times 10^{-4}\ m^3$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{18 \times 10^{-6}}{\frac{4}{3} \times 3.14 \times 9 \times 10^{-4}}$$

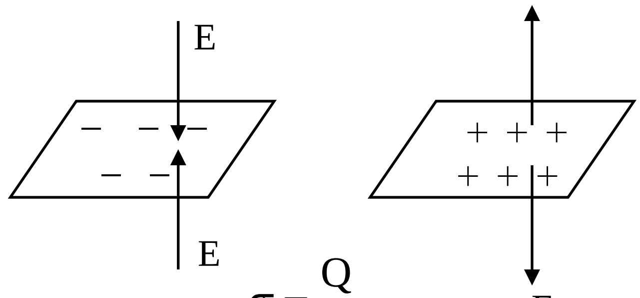
$$= 4.8 \times 10^{-3} \ \text{C/m}^3$$

$$= 4.8 \ \text{mc/m}^3$$

4) Non conducting sheet

لوح غير موصل (وجه واحد فقط مشحون)

$$E = \frac{|\sigma|}{2 \epsilon_0}$$



$$\sigma = \frac{Q}{A}$$

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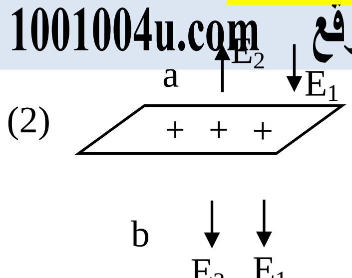
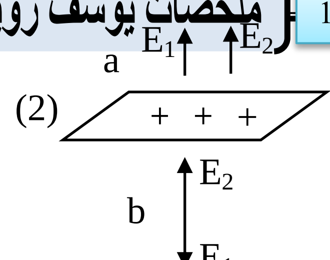
ملحوظة: إذا كان لدينا لوحين متوازيين يمكن حساب E الكلية كما يلي:-

الشحنات متشابهة | **الشحنات مختلفة**





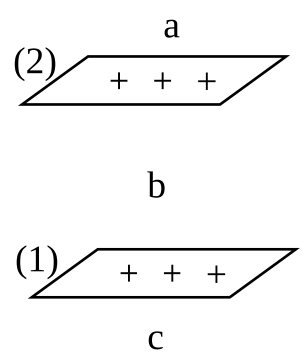
$E_a = \lvert E_1 - E_2 \rvert$	$E_a = E_1 + E_2$
$E_b = E_1 + E_2$	$E_b = \lvert E_1 - E_2 \rvert$
$E_C = \lvert E_1 - E_2 \rvert$	$E_C = E_1 + E_2$

* وفي هذه الحالات نحسب كل E لكل لوح على حدة من العلاقة:

$$E = \frac{|\sigma|}{2\in_0} \qquad \text{and} \qquad \sigma = \frac{Q}{A}$$

Ex:15 Two non-conducting sheets as shown

$\sigma_1 = 2.1\times10^{-5} c/m^2$, $\sigma_2 = 1.4\times10^{-62} c/m$



1) The electric field at a region a is:

a) $3.1\times10^6 N/C$ b) $3.14\times10^6 N/C$

c) $1.28\times10^6 N/C$ d) $2.65\times10^5 N/C$

2) The electric field at a region b is:

a) $1.12\times10^6 N/C$ b) $2.24\times10^{-6} N/C$

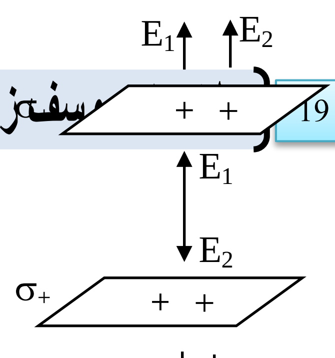
c) $2.42\times10^{-6} N/C$ d) $14.2\times10^6 N/C$

3) The electric field at a region c is:

a) $1.28\times10^6 N/C$ b) $2.56\times10^6 N/C$

c) $13.5\times10^6 N/C$ d) $12.1\times10^6 N/C$

Solutio



$$E_1 = \frac{|\sigma_1|}{2\in_0} = \frac{2.1\times10^{-5}}{2\times8.85\times10^{-12}} = 1.2 \times 10^6 \text{ N/C}$$

$$E_2 = \frac{|\sigma_2|}{2\in_0} = \frac{1.4\times10^{-6}}{2\times8.85\times10^{-12}} = 0.8 \times 10^5 \text{ N/C}$$

(1) $E_a = E_1 + E_2 = 1.28 \times 10^6$ N/C up ward

(2) $E_b = |E_1 - E_2| = 1.12 \times 10^6$ N/C up ward

(3) $E_c = E_1 + E_2 = 1.28 \times 10^6$ N/C down ward

ملحوظة إذا كانت σ سالبة فأن **q** سالبة

Ex:16 Two infinitely non-conducting parallel surfaces carry uniform charge densities of $0.4\text{nc}/\text{m}^2$, $-0.2\text{nc}/\text{m}^2$. The magnitude of the electric field at a point between the two surface is:

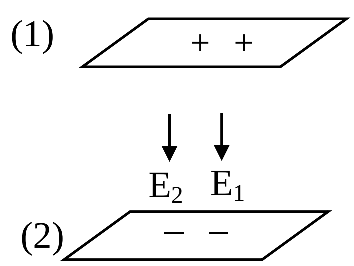
a) 34N/C b) 43N/C

c) 40N/C d) 30N/C

Solution

$\sigma_1 = 0.4\times10^{-9}$ c/m^2 (+)

$\sigma_2 = -0.2\times10^{-9}$ c/m^2 (−)



$$E_1 = \frac{|\sigma_1|}{2\in_0} = \frac{0.4\times10^{-9}}{2\times8.85\times10^{-12}} = 22.6 \text{ N/C}$$

$$E_2 = \frac{|\sigma_2|}{2\in_0} = \frac{0.2\times10^{-9}}{2\times8.85\times10^{-12}} = 11.3\ \ N/C$$

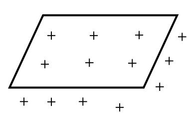
$$E = E1 + E2 = 33.9\ \ N/C$$

$$= 34\ \ N/C$$

5) Conducting sheet (surface)

Metal plate(لوح معدني)

الوجهان مشحونان

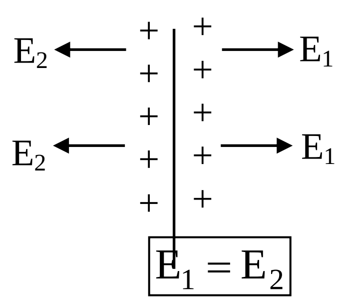


$$E = \frac{|\sigma|}{\in_0}$$

$$\sigma = \frac{Q}{2A}$$

$$Q = 2\sigma.A$$

لأن الوجهان مشحونان



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Ex:17 The electric field just above a thin conducting sheet is 2.5 N/C. The surface charge density of the conductor is:

a) $1.1\times10^{-11} c/m^2$

b) $2.2\times10^{-11} c/m^2$

c) $3.4\times10^{-10} c/m^2$

d) $2.8\times10^{-11} c/m^2$?

Solution

$E = 2.5$ N/C

$\sigma = ??$

$$E = \frac{|\sigma|}{\epsilon_0}$$

$$\sigma = E \cdot \epsilon_0 = 2.5 \times 8.85 \times 10^{-12}$$

$$= 2.2 \times 10^{-11} \quad C/m^2$$

********************The end************************