المعادلات الأسية واللوغاريتمية

$$\log_a x = y \quad \rightarrow \quad x = a^y, \ a \neq 1 \ , \ a, x > 0$$

قاعدة :

❖ اذا كان $a^m = a^n$ فان $m=n$.

❖ اذا كان $x^m = y^m$ بحيث أن $m \neq 0$ عدد فردي، فان $x = y$

مثال

أوجد قيمة x في كل مما يأتي

1. $5^{3x-2} = 125$

$$\rightarrow 5^{3x-2} = 5^3$$

$$\rightarrow 3x - 2 = 3 \quad \rightarrow \quad 3x = 3 + 2 \quad \rightarrow \quad 3x = 5$$

$$\rightarrow x = \frac{5}{3}$$

2. $2^{x-1} = 16$

$$2^{x-1} = 2^4$$

$$\rightarrow x - 1 = 4$$

$$\rightarrow x = 4 + 1 \quad \rightarrow \quad x = 5$$

3. $3(2^x) = 12$

$$\rightarrow 3(2^x) = 3.2^2$$
$$\rightarrow 2^x = 2^2$$
$$\rightarrow x = 2$$

4. $\left(\sqrt[6]{64}\right)^{x+2} = 16$

$$((64^{\frac{1}{6}}))^{x+2} = 4^2 \qquad \rightarrow \qquad (4^{\frac{1}{2}})^{x+2} = 4^2$$

$$\rightarrow 4^{\frac{1}{2}x+1} = 4^2 \qquad \rightarrow \qquad \frac{1}{2}x + 1 = 2$$

$$\frac{1}{2}x = 1 \qquad \rightarrow \qquad x = 2$$

5. $\left(\frac{1}{5}\right)^{7x-2} = (125)^{x-1}$

$\rightarrow (5)^{-7x+2} = (5)^{3(x-1)}$

$5^{-7x+2} = 5^{3x-3}$

$\rightarrow -7x + 2 = 3x - 3$

$-7x - 3x = -5$

$-10x = -5$

$x = \frac{1}{2}$

مثال 2

أوجد قيمة x في كل مما يأتي

1. $\log_x 64 = 3$

$$\rightarrow 64 = x^3$$

$$\rightarrow 4^3 = x^3 \qquad \rightarrow x = 4$$

2. $\log_4 x = 3$

$$\rightarrow x = 4^3$$

$$\rightarrow x = 64$$

3. $\log_4 64 = x$

$$\rightarrow 64 = 4^x$$

$$\rightarrow 4^3 = 4^x$$

$$\rightarrow x = 3$$

4. $\log_x 125 = \dfrac{3}{2}$

$$\rightarrow 125 = x^{\frac{3}{2}}$$

$$\rightarrow 5^3 = x^{\frac{3}{2}}$$

$$\rightarrow x = (5^3)^{\frac{2}{3}} \qquad \rightarrow x = 5^2$$

$$\rightarrow x = 25$$

5. $\log_8(x + 5) = \dfrac{1}{3}$

$$\rightarrow x + 5 = 8^{\frac{1}{3}}$$

$$\rightarrow x + 5 = 2$$

$$\rightarrow x = 2 - 5$$

$$\rightarrow x = -3$$

مثال 3

حل المعادلات التالية :

$1. e^{x^2} = e^{16}$

$$\rightarrow x^2 = 16$$

$$\rightarrow x^2 - 16 = 0$$

$$\rightarrow (x - 4)(x + 4) = 0$$

$$\rightarrow x = 4 , x = -4$$

$2. \log_3 x = -2$

$$\rightarrow x = 3^{-2}$$

$$\rightarrow x = \frac{1}{3^2}$$

$$\rightarrow x = \frac{1}{9}$$

3. $3^x = 3^{5x-6}$

$$\rightarrow x = 5x - 6$$

$$\rightarrow x - 5x = -6$$

$$\rightarrow -4x = -6$$

$$\rightarrow x = \frac{-6}{-4} = \frac{3}{2}$$

$$\square x = \frac{3}{2}$$

4. $\log_x 27 = 3$

$$27 = x^3$$

$$3^3 = x^3$$

$$\rightarrow x = 3$$

5. $5^{3x} = 5^{4x-2}$

$$3x = 4x - 2$$

$$4x - 3x = 2$$

$$x = 2$$

6. $\log_2(x - 7) = 3$

$$x - 7 = 2^3$$
$$x - 7 = 8$$
$$x = 8 + 7$$
$$\rightarrow x = 15$$

7. $25^{x+3} = 125^x$

$$\rightarrow 5^{2(x+3)} = 5^{3x}$$
$$\rightarrow 2(x + 3) = 3x$$
$$\rightarrow 2x + 6 = 3x$$
$$\rightarrow 6 = 3x - 2x$$
$$\rightarrow x = 6$$

8. $\log 5 + \log x = 2$

$$\rightarrow \log(5x) = 2$$

$$\rightarrow 5x = 10^2$$

$$\rightarrow \frac{5x}{5} = \frac{100}{5}$$

$$\rightarrow x = \frac{100}{5}$$

$$\rightarrow x = 20$$

9. $\mathbf{5^3 = (x + 2)^3}$

$$\rightarrow 5 = x + 2$$

$$\rightarrow x = 5 - 2$$

$$\rightarrow x = 3$$

10. $\log(x + 5) = \log(2x - 7)$

$$\log(x + 5) - \log(2x - 7) = 0$$

$$\log\left(\frac{x + 5}{2x - 7}\right) = 0$$

$$\frac{x + 5}{2x - 7} = 10^0$$

$$\frac{x + 5}{2x - 7} = 1$$

$$x + 5 = 2x - 7$$

$$2x - x = 5 + 7$$

$$\square\ x = 12$$

11. $\left(\frac{1}{3}\right)^{x+4} = \left(\frac{1}{3}\right)^{(3x-5)}$

$$x + 4 = 3x - 5$$

$$3x - x = 4 + 5$$

$$2x = 9$$

$$x = \frac{9}{2}$$

$$\square\ x = 4.5$$

12. $\log(x + 1) - \log(2x - 7) = 0$

$$\log(\frac{x + 1}{2x - 7)} = 0$$

$$\frac{x + 1}{2x - 7} = 10^0$$

$$\frac{x + 1}{2x - 7} = 1$$

$$x + 1 = 2x - 7$$

$$2x - x = 1 + 7$$

$$x = 8$$