Compiled by Mostafa Zahri, Ph.D and Hisham Rafat, Ph.D

Introduction to Mathematics



Taibah University, Preparatory Year Program

MATH 101



EARSO

Mathematics

ALWAYS SZARNING

PEARSON

1.3 Polynomials

- Rules for Exponents
- Polynomials
- Addition and Subtraction
- Multiplication
- Division

Rules of Exponents

Rule	Math notation	Description
Product rule	$a^{\mathrm{m}} \cdot a^{\mathrm{n}} = a^{\mathrm{m+n}}$	When multiplying powers of like bases, keep the base and add the exponents.
Power rule 1	$(a^{\mathrm{m}})^{\mathrm{n}} = a^{\mathrm{mn}}$	To raise a power to a power, multiply the exponents.
Power rule 2	$(ab)^{\mathrm{m}} = a^{\mathrm{m}}b^{\mathrm{m}}$	To raise a product to a power, raise each factor to that power.
Power rule 3	$\left(\frac{a}{b}\right)^{\mathrm{m}} = \frac{a^{\mathrm{m}}}{b^{\mathrm{m}}} b \neq 0$	To raise a quotient to a power, raise the numerator and the denominator.
Zero exponent	$a^0 = 1$ $a \neq 0$	A nonzero number to the power of zero equals 1.

Examples: Find each product : a) y^4 . y^7

$$b)(6z^5)(9z^3)(2z^2)$$

Homework1

Simplify:

a)
$$(5^3)^2$$

b)
$$(3^4x^2)^3$$

$$c)(\frac{2^5}{b^4})^3$$

$$d)(\frac{-2m^6}{t^2z})^5$$

Homework 1: Simplify:

a)
$$(5^3)^2$$

b)
$$(3^4x^2)^3$$

c)
$$(\frac{2^5}{b^4})^3$$

d)
$$(\frac{-2m^6}{t^2z})^5$$

Example 2 : Evaluate each power

a)
$$4^{0}$$

$$(-4)^0$$

c)-
$$4^{0}$$

$$(d) - (-4)^0$$

$$e)(7r)^{0}$$

Evaluate $[(-4)^0]^5$

- A. 5
- B. -5
- C. 1
- D. -1

Simplify $\frac{(x^3y^3)^5}{x^5y^2}$.

- A. $x^{3}y^{6}$ B. $x^{2}y$ C. $x^{10}y^{11}$ D. $x^{10}y^{13}$

Simplify $(-3^3)^3$

- A. 3⁶
- B. -3⁹
- C. 3⁹
- D. -3⁶

Algebraic expression.

Any collection of numbers or variables joined by the basic operations of addition, subtractions multiplication or division and so on

$$\left[-2x^2+3x,\frac{15y}{2y-3},\sqrt{m^3-64},(3a+b)^4\right]$$

Term:

The product of a real number and one or more variables raised to powers

Example:

1) the term $-3 m^4$

The coefficient is -3, the variable is m the power (degree) is 4

2) the term $-p^2$

The coefficient is -1, the variable is p the power (degree) is 2

Like Terms:

Are terms with the same variables each raised to the same powers Example:

1) the terms $-3m^4$, $6m^4$, $4m^4$ are like terms

2) the terms $-3y^4$, $6m^4$, $4r^4$ are unlike terms.

Types of Polynomials:

- 1. One term is called Monomial. $-10r^6s^8$
- 2. Two terms is called Binomial. $29x^{11} + 8x^{15}$
- 3. Three terms is called Trinomial. $9p^7 4p^3 + 8p^2$
- 4. More than three terms is called None of These

$$5a^3b^7 - 3a^5b^5 + 4a^2b^9 - a^{10}$$

Which expression is *not* a polynomial?

A.
$$x^3 - 2x^2 + 3x - 2$$

B.
$$-3x + 5x^{14} - 3$$

$$C. x^{-2} + 2$$

D. 5

What is the degree of this polynomial?

$$4x^5 - 5x^4 - 3x^2 + 2$$

- A. 2
- B. 3
- C. 4
- D. 5

The expression $x^3 - \sqrt{2}$ is a polynomial.

A. True

B. False

Addition and Subtraction

Example 3: Adding and subtracting polynomials:
 Add or subtract, as indicated.

• a)
$$(2y^4 - 3y^2 + y) + (4y^4 + 7y^2 + 6y)$$

•b)
$$(-3m^3 - 8m^2 + 4) - (m^3 + 7m^2 - 3)$$

c)
$$(8m^4p^5 - 9m^3p^5) + (11m^4p^5 + 15m^3p^5)$$

d)
$$4(x^2 - 3x + 7) - 5(2x^2 - 8x - 4)$$

Multiplication of Polynomials

- There are several methods for multiplying polynomials.
- •The choice of method depends on the type of polynomials being multiplied together.
- •One of the easiest methods of multiplying polynomials is to use the concept of distribution property.

$$-3x (4x^2 - x + 10) = -12x^3 + 3x^2 - 30x$$

For example: :(Product horizontal)

$$(3x - 4)(2x^{2} - 3x + 5) =$$

$$= (3x - 4)(2x^{2}) - (3x - 4)(3x) + (3x - 4)(5)$$

$$= 3x(2x^{2}) - 4(2x^{2}) - 3x(3x) - (-4)(3x) + 3x(5) - 4(5)$$

$$= 6x^{3} - 8x^{2} - 9x^{2} + 12x + 15x - 20$$

$$= 6x^{3} - 17x^{2} + 27x - 20$$

For example:(Product vertically)

$$(2x^2 - 3x + 5)$$
$$(3x - 4)$$

$$-8x^{2} + 12x - 20 \qquad \leftarrow (-4)(2x^{2} - 3x + 5)$$
$$6x^{3} - 9x^{2} + 15x \qquad \leftarrow (3x)(2x^{2} - 3x + 5)$$

$$6x^3 - 17x^2 + 27x - 20$$

Homework 3:

Multiplication of Polynomials

Multiply
$$(3p^2 - 4p + 1)(p^3 + 2p - 8)$$

FOIL method: (First, Outside, Inside, Last)

Example 4:

Find each product:

a)
$$(6m + 1)(4m - 3)$$

b)
$$(2x + 7)(2x - 7)$$

c)
$$r^2(3r+2)(3r-2)$$
.

Special product

Product of the sum and difference of two terms:

$$(x + y)(x - y) = x^2 - y^2$$

Square of a binomial:

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

Special product

Homework 4: Using the special product

Find each product:

a)
$$(3p+11)(3p-11)$$

b)
$$(5m^3-3)(5m^3+3)$$

c)
$$(9k-11r^3)(9k+11r^3)$$

d)
$$(2m+5)^2$$

e)
$$(3x-7y^4)^2$$

Special product

Example 5: Multiplying more complicated Binomials Find each product.

a)
$$[(3p-2)+5q][(3p-2)-5q]$$

$$b)(x+y)^3$$

$$c)(2a+b)^4$$

Division

Homework 5: Dividing Polynomials

Divide $4m^3 - 8m^2 + 5m + 6$ by 2m - 1.

Division

Example 6: Dividing Polynomials with Missing Terms

Divide $3x^3 - 2x^2 - 150$ by $x^2 - 4$.