• 2.3 - **Quadratic Equations**

- Solving a Quadratic Equation
- Completing the Square
- The Quadratic Formula
- Solving for a Specified Variable
- The Discriminant

Quadratic Equation in One Variable

An equation that can be written in the form

$$ax^2 + bx + c = 0$$

where *a*, *b*, and *c* are real numbers with $a \neq 0$, is a **quadratic equation.** The given form is called **standard form.**

Second-degree Equation

A quadratic equation is a **second-degree equation**—that is, an equation with a squared variable term and no terms of greater degree.

$$x^2 = 25, \quad 4x^2 + 4x - 5 = 0, \quad 3x^2 = 4x - 8$$

Zero-Factor Property

If a and b are real numbers with
$ab = 0$, then $a = 0$ or $b = 0$
or both equal zero.

Example 1

USING THE ZERO-FACTOR PROPERTY

Solve $6x^2 + 7x = 3.$

Square Root Property

A quadratic equation of the form $x^2 = k$ can also be solved by factoring.

$$x^2 = k$$

$$x^2 - k = 0 \quad \text{Subtract } k.$$

$$\left(x - \sqrt{k}\right)\left(x + \sqrt{k}\right) = 0 \quad \text{Factor.}$$

$$x - \sqrt{k} = 0 \quad \text{or} \quad x + \sqrt{k} = 0 \quad \text{Zero-factor property.}$$

$$x = \sqrt{k} \quad \text{or} \quad x = -\sqrt{k} \quad \text{Solve each equation.}$$

Square Root Property

If $x^2 = k$, then

$$x = \sqrt{k} \quad or \quad x = -\sqrt{k}.$$

Square-Root Property

That is, the solution set of $x^2 = k$ is

$\left\{ \sqrt{k}, -\sqrt{k} \right\}$, which may be abbreviated $\left\{ \pm\sqrt{k} \right\}$.

Both solutions are real if $k > 0$,
and both are pure imaginary if $k < 0$.
If $k < 0$, we write the solution set as

$$\left\{ \pm i\sqrt{|k|} \right\}.$$

If $k = 0$, then there is only one distinct solution, 0, sometimes called a **double solution**.

Homework1

USING THE SQUARE ROOT PROPERTY

Solve each quadratic equation.

(a) $x^2 = 17$

(b) $x^2 = -25$

(c) $(x-4)^2 = 12$

Solving a Quadratic Equation by Completing the Square

To solve $ax^2 + bx + c = 0$, where $a \neq 0$, by completing the square, use these steps.

Step 1 If $a \neq 1$, divide both sides of the equation by a.

Step 2 Rewrite the equation so that the constant term is alone on one side of the equality symbol.

Step 3 Square half the coefficient of x, and add this square to each side of the equation.

Step 4 Factor the resulting trinomial as a perfect square and combine like terms on the other side.

Step 5 Use the square root property to complete the solution.

Example 2

USING COMPLETING THE SQUARE ($a = 1$)

1- $x^2 - 4x - 14 = 0.$

2- $9x^2 - 12x + 9 = 0.$

The Quadratic Formula

If we start with the equation $ax^2 + bx + c = 0$,

for $a > 0$, and complete the square to solve for x in terms of the constants a, b, and c,

the result is a general formula

for solving any quadratic equation.

Quadratic Formula

The solutions of the quadratic equation

$ax^2 + bx + c = 0,$

where $a \neq 0,$

are given by the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Caution ***Remember to extend the fraction bar in the quadratic formula extends under the –b term in the numerator.***

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3

USING THE QUADRATIC FORMULA (REAL SOLUTIONS)

Solve $x^2 - 4x = -2$.

Homework 3

USING THE QUADRATIC FORMULA (NONREAL COMPLEX SOLUTIONS)

Solve $2x^2 = x - 4.$

Cubic Equation

The equation $x^3 + 8 = 0$ is called a **cubic equation** because the greatest degree of the terms is 3.

Example 4

SOLVING A CUBIC EQUATION

Solve $x^3 + 8 = 0.$

Solution

Solving for a Specified Variable

Note,

we took both positive and negative square roots. However, if the variable represents time or length in an application, we consider only the *positive* square root.

Homework 4

SOLVING FOR A QUADRATIC VARIABLE IN A FORMULA

Solve for the specified variable. Use ± when taking square roots.

(a) $A = \frac{\pi d^2}{4}$, for d

SOLVING FOR A QUADRATIC VARIABLE IN A FORMULA

Solve for the specified variable. Use ± when taking square roots.

(b) $rt^2 - st = k \quad (r \neq 0)$, for t

The Discriminant

The Discriminant The quantity under the radical in the quadratic formula,

$b^2 - 4ac$, is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ⟵ Discriminant

The Discriminant

Discriminant	Number of Solutions	Type of Solutions
Positive, perfect square	Two	Rational
Positive, but not a perfect square	Two	Irrational
Zero	One (a double solution)	Rational
Negative	Two	Nonreal complex

Caution ***The restriction on a, b, and c is important.***

For example,

$$x^2 - \sqrt{5}x - 1 = 0$$

has discriminant $b^2 - 4ac = 5 + 4 = 9$, which would indicate two rational solutions if the coefficients were integers.

By the quadratic formula, the two solutions

$$\frac{\sqrt{5} \pm 3}{2}$$

are irrational numbers.

Example 5

USING THE DISCRIMINANT

Determine the number of distinct solutions, and tell whether they are *rational*, *irrational*, or *nonreal complex* numbers.

(a) $5x^2 + 2x - 4 = 0$

(b) $x^2 - 10x = -25$

(c) $2x^2 - x + 1 = 0$